Journal
of
The Franklin Institute
Devoted to Science and the Mechanic Arts

Vol. 190 DECEMBER, 1920 No. 6

# THE CHEMISTRY OF THE EARTH'S CRUST.*

BY

HENRY S. WASHINGTON, Ph.D.

Petrologist, Geophysical Laboratory, Carnegie Institution of Washington.

## INTRODUCTION.

THE term "Crust of the Earth" is a heritage from the days when the interior of the earth was generally conceived to be a "sea of molten rock," at an enormously high temperature, covered by a relatively thin, solid crust of cooled matter. Various cogent reasons, into the consideration of which we cannot enter here, have led to the abandonment of this concept, and we now have reason to hold the following tenets as to the conditions that obtain in the earth's interior:

1. The interior is essentially—or, at least, behaves essentially like—a rigid solid, though possibly a certain amount of viscosity may be granted.

2. It is hot, but of an unknown temperature, and probably increases in temperature toward the centre, with a gradient that is unknown beyond very moderate depths, and that is probably very different in different places.

3. It is of a density greater than that of the "crust," inas-

* Based on a paper presented at a meeting of the Section of Physics and Chemistry held Thursday, March 4, 1920.

[Note.—The Franklin Institute is not responsible for the statements and opinions advanced by contributors to the JOURNAL.]

COPYRIGHT, 1920, by THE FRANKLIN INSTITUTE

much as the mean density of the earth as a whole is about 5.55, while that of the crust is about 2.77, as will be shown later.

4. The earth, as a whole, acts in many respects as a magnet, and as the rocks of the crust in general are not notably magnetic, this may be attributed to the characters or composition of the interior materials.

5. From the study of the propagation of earthquakes, we are led to believe that there is a change in the physical properties at about 0.5 of the radius in depth, the matter below this not transmitting transverse vibrations. Studies on the compressibility of rocks by Adams and Williamson, in the Geophysical Laboratory, indicate that the high density of the interior cannot be explained by compressibility, so that we have reason to think that there is also, toward the centre, a change in actual substance.

6. It has been often suggested, and is more or less commonly believed, from consideration of the density and magnetic character of the earth, and from the composition of many meteorites, that part, at least, of the interior is composed essentially of iron, or of nickel-iron alloy similar to those which constitute the iron meteorites.

Leaving the interior of the earth, for the present, we may concentrate our attention on the outer shell—the so-called "crust"—which is the only portion that is directly open to our study, and which has been compared, with some justice, to a covering of slag or scoria over the interior. In dealing with this, we shall consider only its chemical characters, with, towards the end of the paper, some relations between these and the densities of rocks.

The thickness of this crust is, of course, unknown, probably not uniform, and presumably indeterminate. Following Dr. F. W. Clarke, we may assume, for purposes of computation, an approximate thickness of 10 miles (16 kilometres), this being about the (minimum) aggregate thickness of all known rocks and deposits of the various geological ages that have become exposed to our observation and study, through movements in the crust. Incidentally, it about equals the sum of the highest land elevation and the greatest oceanic depth, though no causal nexus is apparent.

This solid crust is made up almost wholly of igneous rock, that is, rock that has solidified from a hot, liquid ("molten") condition, either as "plutonic" rocks, at different depths beneath the surface, or in the form of lava flows at the surface.

Assuming a thickness of ten miles, Dr. Clarke [1] has estimated the rock composition of the crust to be about as follows:

| | | |
|---|---|---|
| Igneous rocks | 95.0 | per cent. |
| Shales | 4.0 | per cent. |
| Sandstones | 0.75 | per cent. |
| Limestones | 0.25 | per cent. |
| | 100.00 | |

Such masses as coal beds, or salt and ore deposits, are of negligible magnitude in studying the chemistry of the crust as a whole, as it is purposed to do here, though their presence is of some significance. The amount of the coating of soil is absolutely negligible from this point of view.

When we take into consideration the oceans and the atmosphere, Clarke estimates the lithosphere at 93 per cent., the hydrosphere at 7 per cent., and the atmosphere at 0.03 per cent., of the complex crust. In the following pages, however, the hydrosphere and the atmosphere, and the sedimentary rocks will not be taken into account, and we shall consider the "crust" as made up wholly of igneous rocks. This is the more justifiable, for our present purposes, because the material of the sedimentary rocks has been derived entirely, either directly or indirectly, from preëxisting igneous rocks, while the metamorphic rocks (gneisses, schists, etc.,) have been formed from either igneous or sedimentary rocks.

When we consider, then, only the igneous rocks of the earth, as a whole, we know that they are not all alike, but show wide differences in their characters, chemical and physical. There are here two main questions regarding them to be considered.

The first is: What is the average chemical composition of the igneous rocks of the crust? The answer to this is of considerable importance for the investigation of the constitution of the earth, and is also of interest in the study of the rocks themselves—the science of petrology.

The second question is: Do the igneous rocks, taken as a whole, show sensible uniformity as to general characters, or do they differ noticeably in different portions of the earth's surface? That is, is the earth's crust sensibly alike or unlike?

[1] Clarke, F. W.: "The Data of Geochemistry." *U. S. Geol. Survey, Bull.* No. 695, p. 33, 1920. The proportion of igneous rock would be still greater with greater assumed thickness of crust.

Attempts to answer these questions, with some consequences that seem to follow from their consideration, will form the chief topics of this paper.

### GENERAL CHARACTERS OF IGNEOUS ROCKS.

For present purposes we cannot go deeply into the characters of igneous rocks, nor discuss them all—a subject that has produced a very voluminous literature. It is needful here to present only some of the salient and pertinent facts.

Igneous rocks, as has been said, are those that have solidified from a state of fusion, or rather liquidity, as the term fusion implies a previous solid condition. The liquid matter, that eventually solidifies as a rock, is called technically the "magma"—a term that is in frequent use in petrology.

The magma comes up from below; from what depth we do not know, though there is some reason for thinking that the places of origin are not very deep. Nor do we know whether it arises from the melting of portions of the earth that are actually solid but potentially liquid on relief of pressure, or whether it is, in general, derived from "reservoirs" of liquid magma.

The igneous magma may be compared, as it usually is, to a complex solution of salts in water. This idea, which was first suggested by Bunsen in 1861, is of great importance, and has been very fruitful in our study of the origin, formation, and characters of igneous rocks.

Among other things, it may be mentioned here that the magma contains various gases in solution, much as air is present in solution in spring water, or, rather more appropriately, as carbon dioxide is present in the waters of many mineral springs, so that it escapes on relief of pressure.

Of these gases, by far the most important, and generally the most abundant, is water vapor. This forms the major part of the clouds that are given off during volcanic eruptions, and, with other gases, produces the sponge-like structure of pumice and the cavities of vesicular lavas through expansion, caused by relief of pressure on reaching the surface. In some glassy lavas water is present to the extent of several per cent., the magma having solidified so rapidly as not to permit of its escape, and inclusions of visible water and liquid carbon dioxide are present in the crystals of many granites and other rocks. The presence of water in

volcanic magmas has been doubted by Brun and others following him, but its existence in lavas, especially those of Kilauea, has been shown conclusively by the researches of Day and Shepherd,[2] is shown by practically every rock analysis, and in other ways, so that the existence of water in magmas may be regarded as one of the established truths of the chemistry of igneous rocks.

Besides water, other gases are often present in volcanic exhalations, such as carbon dioxide, carbon monoxide, hydrogen chloride, sulphur trioxide and dioxide, hydrogen sulphide, hydrogen fluoride, ammonia, methane and possibly other hydrocarbons, sulphur vapor, hydrogen, nitrogen, or oxygen. The study of these and the bearing of their interreactions on the maintenance, and possibly the partial production, of volcanic heat, is an interesting subject.

The presence of these gases in the magma lowers its solidification point, so that a lava, on coming to the surface, may be, and usually is, liquid at a temperature considerably below the fusing point of the solid rock formed from it, during which solidification much, if not most, of the dissolved gas is lost. Either because of this, or because of the lessened viscosity, or in some other way that is not yet well understood, the gases contained in the magma seem to promote the crystallization of minerals, so that they are often referred to as "mineralizers." These gases also play an important part in the formation of many ore bodies.

The magma on solidification generally forms a mixture of minerals, substances of definite chemical composition and physical characters, just as a solution of salts in water (such as sea water), forms a mixture of crystals of salts and ice on freezing. The exception to this is when the cooling of the magma takes place too rapidly for complete, or (as with the obsidians) any, crystallization, in which case the rock is composed partly or wholly of glass. Such glassy rocks are found only as surface flows.

### MINERAL CONSTITUENTS OF ROCKS.

It is a very important and striking fact that, although about one thousand different minerals are known, yet the number of the different kinds that compose by far the great majority of igneous rocks—certainly over 99 per cent. by weight of these—is very

[2] Day and Shepherd, *Bull. Geol. Soc. Amer.*, xxiv, 573, 1913

small. Indeed, the really important and essential igneous rock-forming minerals number only about a dozen.

These essential minerals are: quartz, silicon dioxide; the feldspars, silicates of alumina and potash, soda, or lime, including the potassic orthoclase, the sodic albite, and the calcic anorthite, with isomorphous mixtures of these; the pyroxenes, metasilicates of calcium, magnesium, and iron, with aluminum or sodium in some cases; the amphiboles, in chemical composition much like the pyroxenes, but differing in crystal form and otherwise; the micas, alumino-silicates, for the most part the potassic muscovite or the potassium-iron-magnesium biotite, both containing hydroxyl; the olivines, orthosilicates of iron and magnesium; nephelite, an orthosilicate of sodium and aluminum; leucite, a metasilicate of potassium and aluminum; magnetite, ferroso-ferric oxide, often containing titanium; and apatite, a phosphate of calcium, containing a little fluorine or chlorine. Magnetite and apatite are present in almost all rocks, but seldom in more than almost negligible amounts.

Other minerals are not infrequently met with in certain types of igneous rocks, such as the silicates sodalite, hauyne, melilite, zircon, and garnet, and the oxides tridymite (a second form of silica), ilmenite, chromite, spinel, corundum, and rutile. But, considering igneous rocks from the standpoint of a study of the whole crust of the earth, these are practically negligible. Igneous rocks, then, in general, and looked at in the broadest way, are constituted almost wholly of a very few silicates of aluminum, iron, calcium, magnesium, sodium, potassium, and hydroxyl, with or without quartz (that is, excess of silica), with small amounts of a phosphate and of iron oxide, and with or without traces of other constituents. It is also to be noted that some of the essential minerals enumerated above (the pyroxenes, amphiboles, micas, olivines, and the magnetites), contain small amounts of manganese and titanium. From such a general survey of the rock-forming minerals, then, we obtain the broad lines of the chemical composition of the earth's crust as a whole.

Another important fact concerning the igneous rock minerals is that, with two exceptions, any one of them may occur in rocks with any one or more of the others. The only exceptions to this are that neither nephelite nor leucite is known to occur along with quartz, and a partial exception is that olivine seldom

occurs with quartz, and never in any large amount. Discussion of this and other relations between the various minerals would lead to a consideration of matters outside of our present scope, and would take us too far afield.

Each rock mineral may be present in very widely varying proportions—from practical totality to complete absence. We know igneous rocks that are composed entirely of quartz (arizonite), feldspar (anorthosite), pyroxene (websterite), amphibole (hornblendite), or olivine (dunite), and almost entirely of nephelite (congressite), leucite (italite), or magnetite (some iron ores). Of the essential rock minerals, only the micas and apatite do not form the whole, or almost the whole, of any igneous rock.

From totality of any one mineral we find rocks that are composed of two minerals, more that are composed of three, and still more that are composed of more than three, and with the widest possible variations in the proportions of almost all, with the exceptions noted above as to the non-coexistence of quartz with nephelite and leucite, and its rarity with olivine.

### CHEMICAL CONSTITUENTS OF IGNEOUS ROCKS.

From what has been said it would appear that the various oxides (in terms of which the chemical composition of rocks is usually formulated) may be present in widely different amounts; and, within limits, this is found to be true. All of the constituent oxides have very considerable quantitative ranges, but these differ much with the different oxides. Their possible or recorded maxima are also very different, though in every case the minimum is reached with complete absence. These ranges and maxima will be stated later, after a brief discussion of the oxides that go to make up the igneous rocks.

Though, as we have seen, most igneous rocks are composed of but few essential minerals, and consequently of but few so-called "major" oxides, yet when we come to study them in detail we find that a very considerable number of different chemical constituents may be present in the different rocks. Altogether, about twenty-three are to be found, and are more or less commonly determined and recorded among the better class rock analyses. Indeed, as has been said by Dr. W. F. Hillebrand, the foremost analyst of rocks, "a sufficiently careful examination of these [igneous] rocks would show them to contain all, or nearly all,

the known elements, not necessarily all in a given rock, but more than anyone has yet found." Proper study, therefore, of the chemistry of igneous rocks, and their chemical analysis, if this be complete as to the determination of all the constituents probably present, is evidently a somewhat complicated matter, and one not without difficulties of various kinds.

From the many chemical analyses of rocks that have been made since this was first attempted very early in the nineteenth century, (the total number of published rock analyses now numbering about 12,000), we have a good idea of what chemical constituents make up rocks, their relative abundance, and their various ranges in percentage.

By far the most important and generally the most abundant are what are called the "major" constituents. These are nine in number and, stated as oxides, are: silica ($SiO_2$), alumina ($Al_2O_3$), ferric oxide ($Fe_2O_3$), ferrous oxide ($FeO$), magnesia ($MgO$), lime ($CaO$), soda ($Na_2O$), potash ($K_2O$), and water ($H_2O$).[3] Together these nine oxides make up about 98 per cent. of igneous rocks, and all of them are present in greater or less amount in practically every rock, so that the amount of each must be determined in every chemical analysis of a rock that makes the slightest pretense to good quality.

As the most abundant and essential rock minerals are either silica or silicates, and as all igneous rocks, with the exception of some rare and small iron ore bodies of magmatic origin, are consequently silicate rocks, silica shows easily the highest maximum and the widest range, both in extremes and in the usual run of occurrence. A few igneous rocks are known that are composed almost entirely of quartz,[4] and the highest silica percentages recorded for igneous rocks are 98.77 and 97.65, in rocks from the Transvaal; while one from Cumberland (England), the border facies of a granitic mass, shows 96.16, one from Massachusetts shows 93.38, and one from Arizona 92.59. In general, however, the percentage of silica ranges from about 75 to about 34, and it drops to zero only in some "magmatic" iron-ore bodies. In almost all rocks it is the most abundant constituent.

[3] This order is not quite that of relative abundance, but that which is commonly used in the statement of rock analyses.

[4] Quartz veins are not considered here, as they are usually of non-igneous origin, at least in the commonly accepted sense.

Alumina, which is almost invariably the next most abundant constituent, reaches a maximum of about 60 per cent. in some corundum-bearing syenites from Canada and the Urals, and has a general range from about 20 to about 10. It is wholly absent only in the "magmatic" ores, and in some rocks that are composed entirely, or almost so, of olivine. The two oxides of iron reach, of course, their maxima in such rocks as the iron ores already spoken of;[5] the highest figures recorded for $Fe_2O_3$ being 88.41 (Sweden) and 62.39 (Ontario), while for FeO they are 34.58 (Sweden) and 32.92 (Minnesota). Their general range is from nearly 15 for each (FeO generally higher than $Fe_2O_3$), and but little more than that for both together in any one rock, to less than one-half of one per cent. Iron is seldom entirely absent.

Magnesia reaches its maximum in the almost purely olivine rocks (dunites) of North Carolina, 48.58, and of New Zealand, 47.38, but its general range is from about 25 to much less than 1 per cent. Lime is highest (22.52) in some pyroxenites of the Urals, and almost as high (about 20) in the anorthosites of Canada and elsewhere, but it ranges in general from about 15 to nearly zero.

Of the two alkalies, soda reaches a maximum of 19.48 in a rare rock from Canada, and of 18.67 in another from Turkestan; but its general range is from about 15 per cent. down to nearly zero. It is hardly ever entirely absent. Potash shows a somewhat smaller range than soda, its maximum being 17.94 in a recently discovered lava from Italy, the next highest figure being 11.91 from Wyoming; but in general it seldom gets above 10 per cent., ranging from that down to zero. Its amount is generally less than that of soda.

As regards water, the last of the major constituents, a few volcanic glasses are known which, although perfectly fresh and undecomposed, contain up to 8 or 10 per cent., and there are some fresh crystalline rocks that contain from 3 to 5 per cent. Generally, however, if a rock contains more than about 2 per cent. of $H_2O$, this can usually be attributed to alteration, though few rocks are quite free from this constituent.

After the major come the "minor" constituents, which are

[5] It is a question whether all of these ore bodies are to be considered as really igneous rocks, though some undoubtedly are.

almost always present in very small amounts, seldom over 2 per cent. for any one, or rarely up to 5 per cent. for all of them, in any one rock. Of these minor constituents, three are of special importance, partly because of their almost constant presence, and partly because they are generally present in largest amount. These three are: titanium dioxide, phosphorus pentoxide, and manganous oxide, and all three should be determined in a good rock analysis.

Titanium dioxide ($TiO_2$) reaches a maximum in some very rare rocks from Virginia (69.67 and 65.90) and Quebec (53.35), but as a general thing its percentage is seldom over 5, and is mostly from about 2 to nearly zero. Of the many rocks of all kinds that I have analyzed, there has not been a single one that did not contain titanium, in some cases in very small, but always in easily determinable, quantity. This is also the experience of Doctor Hillebrand,[8] and probably of every other experienced analyst of rocks.

The maximum for phosphorus pentoxide ($P_2O_5$) is but a little above 16 per cent. in some highly unusual rocks from Sweden and Virginia, that are composed largely of apatite, with titaniferous magnetite or rutile. In few rocks, however, is it above 3 per cent., and its general range is from about 1 per cent. to zero. It does not seem to be present so constantly as titanium (or manganese), as one occasionally meets with a rock that shows no trace of it, though this may be because of the more delicate tests for the other two.

Manganese, as manganous oxide (MnO), is present in practically every rock that has been analyzed, but its maximum is much lower than those of titanium and phosphorus oxides. Some, if not most, of the high figures reported for it are almost certainly due to analytical errors, and the highest recorded figures that are trustworthy are 1.90 and 1.46 in two rocks from Bahia, Brazil. Its general range is from 0.3 per cent. to about zero.

The other minor constituents that are readily determinable, and many of which are indeed determined in good analyses, are quite varied. The list is as follows: Carbon dioxide ($CO_2$), zirconia ($ZrO_2$), chromium sesquioxide ($Cr_2O_3$), vanadium sesquioxide ($V_2O_3$), the "rare earths" ($(Ce, Y)_2O_3$), nickel oxide (NiO), strontia (SrO), baryta (BaO), lithia ($Li_2O$), sulphur as

[8] Hillebrand, W. F.: *U. S. Geol. Survey, Bull.* No. 700, p. 25, 1919.

both sulphide (S) and sulphur trioxide ($SO_3$), chlorine (Cl), and fluorine (F). To these might be added boron, cobalt, copper, glucinum, lead, molybdenum, nitrogen, and zinc, which, however, are almost always present in such extremely small amount, or the analytical difficulties are so great for the separation of the small quantities in which they occur, that their determination is rarely attempted.

The maxima and ranges of some of these may be briefly stated. Carbon dioxide may be present, as a component of a few minerals (as in primary calcite and cancrinite), in some unaltered rocks; but its presence is generally due to alteration. In one calcite trachyte from Spain its amount is 7.69 per cent., and in cancrinite rocks it may reach about 1.70, the carbonate minerals in these being apparently primary. But it is generally considered as a measure of the alteration of the rock by weathering, etc.

Zirconia is much less abundant than the closely related titania and, though it reaches a maximum of nearly 5 per cent. in some Greenland rocks, in general it seldom is over 1 per cent., is usually much less, and is quite absent from most rocks. It forms, by the way, one of the most striking illustrations of the correlation of the occurrence of different elements in different kinds of rocks, as will be brought out later.

Baryta and strontia are very commonly present, though they are seldom determined in analyses made outside of the United States, Canada, and Australia. In almost every case the amount of baryta is much greater than that of strontia, this being an exception to a general rule as to the occurrence of related elements, to be mentioned later. They both reach their maxima in certain exceptional, highly potassic rocks of Wyoming, of about 1 per cent. for baryta and 0.3 for strontia; though usually baryta is present up to but a few tenths of one per cent., and strontia in hundredths.

Sulphur, as sulphides, is present up to about 9 per cent. in a peculiar, pyrrhotite-bearing rock from Maine, and probably in similar amounts in some sulphide ore-bodies of magmatic origin in Norway, which have not been fully investigated. But, as a rule, its amount is seldom over 1 or 2 per cent., and is usually in tenths of a per cent. The highest figures for sulphur trioxide are about 2.5 per cent. in rocks from Apulia and Kamerun, and somewhat lower on Tahiti, but these are exceptional, and it is usually present only in tenths or hundredths of a per cent. Much

the same can be said of chlorine, the highest figures for which are those of a rock from Turkestan (about 7), one from Quebec (4.47), and one from French Guinea (2.80). It is present in many rocks, especially lavas, but only in a few tenths or hundredths of a per cent.

Chromium sesquioxide is known to be present up to about 4 per cent. in some ores from Greece, which are probably of magmatic origin, and is reported as between 2 and 3 per cent. in some undoubtedly igneous rocks from Baden. But these are highly exceptional, and about 0.5 may be taken as its usual maximum. It is generally entirely absent. Vanadium sesquioxide is always present in much less quantity and is usually quite absent. The oxides of the rare earth metals, chiefly ceria and yttria, reach a maximum of 1.79 in a rare type of rock from Madras, 0.6 in one from Sweden, and 0.4 in one from the islet of Rockall, but the usual maximum is only one- or two-tenths of 1 per cent. They are less often determined than they should be. Nickel oxide is present in some rocks up to about 0.2 per cent. The maximum amount of each of the other minor constituents may be placed at not over 0.5 per cent., and they are almost always found only as one- or two-tenths, or still more often as hundredths, of a per cent., or are absent. Indeed, for most of the minor constituents the quantities usually yielded by analysis are so small as to be significant only as to their actual presence or absence.

A few words may be said of boron, glucinum (beryllium), and scandium, as these enter into a later phase of the subject. The analytical difficulties involved in their determination, for the extremely small amounts that are present, are so great that the percentage of none of these is recorded for any rock. Yet they are all known to be rather widely distributed among the igneous rocks, boron in tourmaline, glucinum in beryl, and both in some other rarer minerals, while the widespread occurrence of scandium among igneous rocks, though in very small amounts, has been shown spectroscopically.[7]

### THE AVERAGE IGNEOUS ROCK.

We come now to the consideration of the average chemical composition of the earth's crust, that is, of all igneous rocks. Apparently Dr. F. W. Clarke was the first to undertake this

[7] Eberhard, C.: *Sitzb. kg. preuss. Akad. Wiss.*, 1908, p. 851.

estimation,[8] basing his conclusions largely on the numerous analyses that had been made by the chemists of the United States Geological Survey. Since then he and others, Harker, Mennell, Knopf, Mead, and the writer, have published other estimates, which, it may be said here, do not differ greatly the one from the other. The latest discussion of this subject is to be found in the last edition of Clarke's "Data of Geochemistry,"[9] where numerous references to the literature are given.

The true estimation of the average chemical composition of the igneous rocks is by no means such a simple matter as it may appear to be at first thought, and, before we deal with it, it will be as well to state very briefly some of the disturbing factors that are involved. The matter will be treated in greater detail in a forthcoming paper by Doctor Clarke and the writer.

In the first place, we know but little of the exact chemical characters of the igneous rocks of many districts of the earth. This is true of the great continents of Asia and South America, as well as of Africa and Australia, in all of which we have, for the most part, a knowledge only of the rocks more or less near the coasts, and know only in a general and very imperfect way the rocks that constitute the vast expanses of the interior portions. The same ignorance, either total or partial, holds true for many countries, such as China, Arabia, and even Brazil, India, Egypt, and Spain, in which the number of analyses is quite disproportionate to the number and masses of igneous rocks that are known to occur. A most striking example is furnished by the West Indies, where, of the igneous rocks of the otherwise well-known and readily accessible large islands of Cuba, Jamaica, Porto Rico, and Haiti, we do not possess a single analysis.

Most of the countries of Europe are well represented, but for the most part with not very complete analyses. North America is well known, especially as to the rocks of the United States and southern Canada. The analyses of both these countries are of exceptionally high general quality. Parts of Australia, especially New South Wales, Victoria, and Queensland, with New Zealand, are well represented, as is also British Guiana, and it should be said that the analyses of Australian and British Guiana rocks are

[8] Clarke, F. W.: *Bull. Phil. Soc. Wash.*, xi, p. 131, 1889; also *U. S. Geol. Survey, Bull.* 78, p. 34, 1891.

[9] Clarke, F. W.: *U. S. Geol. Survey Bull.*, 695, pp. 24 ff., 1920.

almost the only ones that, as a whole, are comparable as to accuracy and completeness with those of the United States, which holds a preëminent position through the labors of the chemists of the United States Geological Survey.

A second disturbing factor, and one that has been often advanced against the validity and representativeness of the estimates of the average composition of rocks, is that the true relative amounts of various rocks are not properly represented because of the selection of material for analysis. It has frequently happened that the petrographer has had analyzed rather the rare or most interesting rock types than those which, though much more abundant in the region described, are of more usual character. While this is often to be expected and, from a special point of view, is almost justifiable, yet it certainly may involve a serious disturbance in the estimation of the composition of the crust as a whole. This is so, because the most interesting types, often *ipso facto*, are much less abundant than the common ones, so that, as regards the relative masses of the various kinds of rocks in any given region, they are disproportionately represented. It is needless here to give examples, of which there are very many; it would lead us too far into the technicalities of petrography.

Although this objection is serious, and is entitled to consideration, yet it would seem, on detailed examination, not to be of the overwhelming character that is often attributed to it. For one thing, the satellitic rocks of the dikes and other small bodies (which are most prone to furnish "interesting" types), tend to be complementary to each other, through processes of differentiation, and so, as Doctor Clarke says, "they tend to compensation, and so to approximate to the true mean." Also, as in a number of examples from many localities that could be cited, only the main body or the most prominent types have been analyzed, chiefly because of the labor or expense of making chemical analyses of rocks. Again, as I have pointed out elsewhere, the more "basic" rocks, that is, those that are lowest in silica and highest in iron oxides, magnesia, and lime, are most liable to alteration, so that many of their analyses would be excluded from the data selected for our purpose, for which only analyses of fresh and unaltered rocks are considered.

These, and other considerations that might be mentioned, tend to minimize the rather prevalent idea that the averages, such

as have been calculated in former years by Doctor Clarke and me, are not strictly representative, in that the well-known apparent preponderance of granitic rocks is not sufficiently emphasized. Attempts have been made by some to correct such errors by weighting the average analyses of the various rock types by their areal values.[10] Such a procedure, however, is open to two objections: As much weight is thus allowed for lava flows, of manifestly small vertical extension, as for massive intrusive bodies presumably of much greater depth; and, as Clarke points out, "the surface exposure of a rock is no certain measure of its real volume and mass, for it may be merely the peak or crest of a large formation."

But the serious objection to any such attempts at correcting what may be, and often admittedly are, defects in our data, is that they introduce unduly the personal equation, and thus may, or are likely to, introduce other errors of unknown and indeterminate magnitude. As has been shown very briefly above, we are as yet in great ignorance as to the igneous rocks of a large portion of the earth's surface and crust, and it would seem to be the philosophical attitude to admit this and, as Doctor Clarke[11] says, "do the best we can with the available data." They are admittedly not ideal, but an attempt to better them, at this stage of our knowledge, is more likely than not to make a "bad matter worse." Let us be philosophical Italians for a moment, and say with them "Ci vuol pazienza."

Apart from such fundamental considerations of the character of our basal data as have been all too briefly touched on above, we meet with others when we come to consider the analyses themselves. No analyses are ideally perfect, either as to accuracy or completeness, but, while it is obviously the desirable procedure to exclude from our data rock analyses that may not be up to the ideal mark that we may set, yet, by so doing, we shall inevitably reduce the number of our data so as probably to more than offset their excellence in quality. We should have and use, of course, only analyses that are perfectly accurate and complete as to the determination of all the constituents that may be present. But—"humanum est errare," and so we must here also "do the best we

[10] Daly, R. A.: "Igneous Rocks and Their Origin," New York, 1914, pp. 19–46, 168–170; and Knopf, A.: *Jour. Geol.*, xxii, p. 772, 1914.

[11] Clarke, F. W.: *Proc. Amer. Phil. Soc.*, li, p. 215, 1912.

can with the available data," excluding, of course, from consideration analyses that are manifestly bad. Consideration of this topic would lead us too far astray, but it will be found discussed elsewhere.[12]

It may also be mentioned here that, as some of the minor constituents are, in the course of analysis, precipitated and weighed with others, and are later determined separately and subtracted from the previous total figure, if these are not determined the figure for the main constituent will be too high. This is notably the case with alumina, with which are precipitated and weighed the oxides of titanium, phosphorus, rare earth metals, zirconium, chromium, and vanadium, with often some manganese. If the analysis is not complete as regards these constituents, therefore, the figure for alumina will be too high.

As has been said above, the average composition of the igneous rocks has been estimated by several petrologists—Clarke, Harker, Loewinson-Lessing, Daly, Knopf, Mead, and myself. Clarke based his earlier estimates very largely on the analyses of rocks from the United States, as did Knopf, while Harker's average was of rocks from Great Britain alone. In his latest estimates Clarke included rocks from all over the globe, as did I in my own computation. This also was the basis of Daly's and Mead's computations, though in both their estimates, which were founded largely on personal selection of what constituted "types" of various rocks, the personal equation enters somewhat unduly. As we shall see later, continental averages, or others selected from regional data, differ too much to be representative of the average composition of the whole "crust."

The basis for the present, and latest, estimate was the collection of rock analyses that has recently been published.[13] This includes practically all the analyses of igneous rocks, from all over the earth, that have been published between 1883 and 1913, inclusive. These amount to 8602 analyses, of which 5179 of fresh rocks were considered to be "superior;" that is, satisfactory as to accuracy and completeness. Only these 5179 analyses were used. The computations of the various averages, for the whole earth, the continents, and various districts of the earth's surface, were made by Dr. F. W. Clarke, during the sum-

[12] Washington, H. S.: *U. S. Geol. Survey, Prof. Paper* No. 99, pp. 10–26, 1917.

[13] Washington, H. S.: *U. S. Geol. Survey, Prof. Paper* No. 99, 1917.

mer of 1919. To him I am greatly indebted for his very painstaking and laborious undertaking, and would express my great appreciation of his kindness in permitting the present publication of some of his results. It must be said that there are presented here only a few of these, and that all the data in detail, with certain considerations of them, are to be published by us jointly in the near future, as a Professional Paper of the United States Geological Survey.

TABLE I.

*Average Composition of the Earth's Crust.*

| | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| Silicon dioxide ($SiO_2$) | 59.09 | 58.59 | 57.78 | 59.83 |
| Aluminum sesquioxide ($Al_2O_3$) | 15.35 | 15.04 | 15.67 | 15.02 |
| Ferric oxide ($Fe_2O_3$) | 3.08 | 3.94 | 3.31 | 2.62 |
| Ferrous oxide (FeO) | 3.80 | 3.48 | 3.84 | 3.43 |
| Magnesium oxide (MgO) | 3.49 | 4.49 | 3.81 | 3.74 |
| Calcium oxide (CaO) | 5.08 | 5.29 | 5.18 | 4.83 |
| Sodium oxide ($Na_2O$) | 3.84 | 3.20 | 3.88 | 3.37 |
| Potassium oxide ($K_2O$) | 3.13 | 2.90 | 3.13 | 3.05 |
| Water ($H_2O$) | 1.14 | 1.96 | 1.76 | 1.90 |
| Titanium dioxide ($TiO_2$) | 1.05 | 0.55 | 1.03 | 0.79 |
| Phosphorus pentoxide ($P_2O_5$) | 0.30 | 0.22 | 0.37 | 0.29 |
| Manganous oxide (MnO) | 0.125 | 0.10 | 0.22 | 0.10 |
| Carbon dioxide ($CO_2$) | 0.102 | 0.37 | .. | 0.49 |
| Zirconium dioxide ($ZrO_2$) | 0.039 | .. | .. | 0.023 |
| Sulphur (S) | 0.053 | 0.034 | .. | 0.10 |
| Chlorine (Cl) | 0.056 | 0.012 | .. | 0.063 |
| Fluorine (F) | 0.078 | .. | .. | 0.10 |
| Chromium sesquioxide ($Cr_2O_3$) | 0.056 | 0.021 | .. | 0.048 |
| Vanadium sesquioxide ($V_2O_3$) | 0.032 | .. | .. | 0.026 |
| Nickelous oxide (NiO) | 0.025 | .. | .. | 0.026 |
| Barium oxide (BaO) | 0.055 | 0.033 | .. | 0.10 |
| Strontium oxide (SrO) | 0.022 | 0.009 | .. | 0.043 |
| Lithium oxide ($Li_2O$) | 0.007 | 0.011 | .. | 0.011 |
| | 100.000 | 100.250 | 100.00 | 100.000 |

1. Latest estimate, Clarke and Washington, 1920.
2. F. W. Clarke (first estimate). *Bull. Phil. Soc.*, Wash., xi, p. 135, 1889.
3. Washington, *Prof. Paper* No. 99, p. 108, 1903.
4. Clarke, *U. S. G. S., Bull.*, No. 695, p. 28, 1920.

In Table I there is given the most recent calculation of the average igneous rock, together with three of the most important of the earlier estimates. As regards No. 1, it is to be noted that the figures for the main constituents, from silica to water, inclusive, were arrived at by dividing the sums total of the various determinations by the whole number of analyses, as all of these constituents were determined in all the analyses used. For the

minor constituents, from titanium dioxide to lithium oxide, inclusive, the figures given are the means of the sums total of the various constituents divided by the whole number of analyses and also by the number of determinations. The former would presumably give a minimum, and too low an average, and the latter would probably be too high; while the mean would be probably rather nearer the true figure. This matter has been discussed by Clarke and by me elsewhere, and will be enlarged on further in our joint publication. The figures given here should be considered as provisional, as adequate discussion of their relative merits is not called for here. The figure for fluorine is almost certainly too high, as are probably those for chromium, barium, and one or two other oxides, while possibly that for carbon dioxide is a trifle too low.

From No. 1 it will be seen (and the same is approximately true of all the others), that the first nine oxides (from silica to water, inclusive), constitute 98 per cent. of the whole, and that these, with the oxides of titanium, phosphorus, and manganese, make up together 99.475 per cent. of the crust, leaving only a trifle more than one-half of one per cent. for all the other oxides.

Thus we see that in round numbers silica is the most abundant, and constitutes about six-tenths (nearly two-thirds) of the crust; alumina is next—a very poor second—slightly more than one-seventh; then the two iron oxides, together about one-twelfth; lime about one-twentieth; soda about one twenty-fifth; magnesia about one-thirtieth; potash about one thirty-third; water and titanium dioxide each about one one-hundredth; phosphorus pentoxide about one three-hundredth, and manganous oxide about one eight-hundredth, while carbon dioxide is about one one-thousandth. Each of the others is notably less than one one-thousandth. It will be observed that in the list, which includes all the constituents that may be commonly determined in really good and complete analyses of igneous rocks, neither lead, tin, zinc, mercury, silver, gold, platinum, arsenic, antimony, or several other of the elements commonly used in daily life, are represented. The only common metals shown are iron, aluminum, manganese, and nickel. This is a rather important point that will be adverted to later.

In order to form an idea of the actual rock that a magma of this average composition would form under normal conditions,

we must calculate, from the data given by the analysis, the presumable actual mineral composition, or the "mode," as it is technically called. There are two general and important conditions controlling the products of solidification that may be considered. The magma may have solidified at considerable depth, slowly and under great pressure; or it may have solidified, as a lava flow, on the surface; that is, rapidly and under low pressure. The former would furnish what is called a plutonic rock (as a granite or a gabbro), and the latter an effusive one (as a rhyolite or a basalt); and the different conditions of solidification would bring about certain changes in the mineral composition of the resulting rock.

Such a calculation leads to the following results, which are to be considered as only approximately correct, as variations in the mode, of slight extent but in different directions, may be brought about by slight variations in the conditions of solidification. As a plutonic rock the magma would form a so-called granodiorite; that is, a rather coarse-grained, holocrystalline rock, much like many granites (and which would be commonly called a rather dark granite), composed of feldspar, quartz, hornblende or biotite, and very small amounts of magnetite and apatite. If it solidified under surface conditions, the magma would form that most common kind of lava, an andesite, rather fine-grained, light gray or pinkish, and showing small crystals ("phenocrysts") of feldspar and either hornblende or pyroxene, with perhaps a little biotite, in a dense "groundmass." Under the microscope the groundmass would show feldspars, pyroxene (or hornblende), and possibly a little quartz, with small grains of magnetite and apatite, and with or without glass, according to the rapidity of cooling.

Stated in quantitative terms of "modal" or actual minerals, the rocks would have probably the following approximate compositions:

| | Granodiorite | Andesite |
|---|---|---|
| Quartz | 11 | 10 |
| Andesine (Na-Ca feldspar) | 47 | 47 |
| Orthoclase (K feldspar) | 16 | 18 |
| Hornblende and biotite | 20 | 19 |
| Pyroxene | | |
| Magnetite | 5 | 5 |
| Apatite | 1 | 1 |

It will be seen that, in either case, the average rock would be composed entirely of the most common minerals, as is to be expected, with the exceptions of olivine, nephelite, and leucite, which are much less often met with and which, furthermore, are not found in rocks with an excess of silica, as is true of the average rock. Inasmuch as the average rock would have been formed at some depth beneath the surface, the average crust may be considered to be a granodiorite, with the general characters and mineralogical composition described briefly above. The fact, however, must not be lost sight of that locally the crustal rock may vary considerably, as will be pointed out later. We are dealing here only with the average of the crust as a whole.

We may examine the chemical composition of the earth's crust in greater detail and, as has been done by Clarke in the papers cited above, reduce the figures of the analysis to the form of the component elements. The results are given in the annexed Table II, there being here presented, not only the elementary constituents of the average rock given in Table I, but in addition data showing the relative abundance of some other of the more important elements that are not usually, or indeed are never, determined in the analysis of rocks. The data for these are taken from estimates by Vogt, De Launay, and Kemp, with additional data by Clarke and Steiger for a few, and some additions and changes in relative position based on my own studies. An "x" means a digit in the respective decimal place or places. The elements are presented in their order of relative abundance.

This average, it must be repeated, does not include the sedimentary rocks or contituents of the hydrosphere or of the atmosphere. Clarke has included these in several of his estimates, and his latest shows that the percentage, on this basis, of oxygen is 50.02, of silicon 25.80, of aluminum 7.30, and of the other most abundant elements in similarly slightly less amounts than in Table II. When thus reckoned chlorine and carbon fall in between titanium and phosphorus, with percentages, respectively, of 0.20 and 0.18, while nitrogen appears between chromium and zirconium with a percentage of 0.03.

Leaving these refinements out of consideration here, there are some striking features presented in the table to which attention may be called. The first is the appearance among the abundant elements of some that are usually counted as rare. Among these

TABLE II.

*The Chief Elements in the Earth's Crust in Order of Abundance.*

| | | |
|---|---|---|
| 1. | Oxygen | 46.43 |
| 2. | Silicon | 27.77 |
| 3. | Aluminum | 8.14 |
| 4. | Iron | 5.12 |
| 5. | Calcium | 3.63 |
| 6. | Sodium | 2.85 |
| 7. | Potassium | 2.60 |
| 8. | Magnesium | 2.09 |
| 9. | Titanium | 0.629 |
| 10. | Phosphorus | 0.130 |
| 11. | Hydrogen | 0.127 |
| 12. | Manganese | 0.096 |
| 13. | Fluorine | 0.077 |
| 14. | Chlorine | 0.055 |
| 15. | Sulphur | 0.052 |
| 16. | Barium | 0.048 |
| 17. | Chromium | 0.037 |
| 18. | Zirconium | 0.028 |
| 19. | Carbon | 0.027 |
| 20. | Vanadium | 0.021 |
| 21. | Nickel | 0.019 |
| 22. | Strontium | 0.018 |
| 23. | Lithium | 0.003 |
| 24. | Copper | 0.002 |
| 25. | Cerium, etc. | 0.001 |
| 26. | Glucinum | 0.00xx |
| 27. | Cobalt | 0.00xx |
| 28. | Boron | 0.000x |
| 29. | Zinc | 0.000x |
| 30. | Lead | 0.000xx |
| 31. | Arsenic | 0.000xx |
| 32. | Cadmium | 0.0000xx |
| 33. | Tin | 0.0000xx |
| 34. | Mercury | 0.0000xx |
| 35. | Antimony | 0.0000xx |
| 36. | Molybdenum | 0.0000xx |
| 37. | Silver | 0.00000xx |
| 38. | Tungsten | 0.00000xx |
| 39. | Bismuth | 0.00000xx |
| 40. | Selenium | 0.000000xx |
| 41. | Gold | 0.000000xx |
| 42. | Bromine | 0.000000xx |
| 43. | Tellurium | 0.0000000xx |
| 44. | Platinum | 0.0000000xx |
| | | 100.000 |

are especially titanium, barium, chromium, zirconium, vanadium nickel, strontium, and lithium, with copper, cerium, glucinum, cobalt, and boron among those to which no definite figures can be assigned as yet. Titanium occupies the ninth place among the elements (tenth among the oxides), although it is usually considered a "rare" element, and its name and existence are possibly unknown to many persons. The establishment of this fact, probably a very important one in our study of the constitution of the earth, is due primarily to the chemists of the United States Geological Survey, who, under the leadership of Dr. W. F. Hillebrand beginning in the early eighties of the last century, first began to determine the "rarer" elements in their analyses of the rocks of this country. Similarly, they found that some others of the supposedly rare elements are widely distributed, notably barium, strontium, chromium, vanadium, nickel, and even molybdenum.[14]

In Table II it is also noteworthy that, of the metals in daily and common use, only aluminum, iron, manganese, chromium, vanadium, and nickel, appear among those elements that are present in the rocks of the crust in sufficient amount to be commonly determinable by the usual processes of analysis. Such common and "every-day" metals as copper, zinc, lead, tin, mercury, silver, gold, and platinum, antimony, arsenic, and bismuth—metals that are of the utmost importance to our civilization and our daily needs—all these are to be found in igneous rocks, if at all, only in scarcely detectable amounts. Though they are ultimately derived from the igneous rocks, they are made available for our use only by processes of concentration into so-called ore bodies.

To give some concrete and striking figures, it may be pointed out that the eight most abundant elements of the earth's crust (oxygen, silicon, aluminum, iron, calcium, sodium, potassium, and magnesium)—the only ones whose amounts are over 1 per cent.—constitute together 98.63 per cent. of the crust. These, with titanium, phosphorus, hydrogen, and manganese—twelve in all—make up 99.612 per cent.; thus leaving but about 0.39 per cent. for all the other elements, among them some that are quite indispensable for our existing civilization.

[14] Cf. Hillebrand: *Jour. Amer. Chem. Soc.*, xvi, pp. 81–93, 1894; *Amer. Jour. Sci.*, vi, p. 209 1898; *U. S. Surv. Bull.* 700, p. 24, 1919.

A cursory examination shows that the most abundant elements in the earth's crust are, on the whole, those of low atomic weight, as has been often pointed out, while the rarer ones are, in general, those of higher atomic weight. It has also been pointed out by Clarke[15] that, considering the several elements of any group in the periodic table (for which see page 780), while the first member is comparatively rare, the second is the most abundant (the oxygen group being the only exception), and the members become increasingly rare with increasing atomic weight. This is well seen, for example, in Group 1 (lithium, sodium, potassium, rubidium, and cæsium); in Group 2 (glucinum, magnesium, calcium, strontium, and barium), though here we have inversions of the rule in the relative abundance of magnesium and calcium, and of strontium and barium. It is also seen in the third group (boron, aluminum, scandium, gallium, etc.); in the fourth group (carbon, silicon, titanium, zirconium, and cerium); in the fifth (nitrogen, phosphorus, arsenic, and antimony); in the sixth (oxygen, sulphur, selenium, and tellurium), here again there being an inversion as regards the first and second; and in the seventh (fluorine, chlorine, bromine, and iodine). It is also seen in Group 8, in the case of iron, nickel, and cobalt, according to their atomic weights, though the atomic numbers of nickel and cobalt are reversed in order. As Clarke says: "We are dealing with an evident tendency of which the meaning is yet to be discovered. That the abundance and associations of the elements are connected with their position in the periodic system seems, however, to be clear. The coincidences are many, the exceptions are comparatively few."

The relation of the abundance of the elements to the periodic law has also been discussed recently by Harkins,[16] who holds that the abundance of the elements is "related to the atomic number and not to the periodic system," that the abundant elements are those of low atomic weight with an atomic number less than 29, and that the elements with even-numbered atomic numbers surpass in abundance the odd-numbered. It would take us too far from our proper subject to discuss this very interesting topic here, but we may examine briefly a hitherto unrecognized phase

[15] Clarke, F. W.: "Data of Geochemistry," *U. S. Geol. Surv. Bull.* 695, p. 39, 1920.

[16] Harkins, W. D.: *Jour. Amer. Chem. Soc.*, xxxix, p. 856, 1917.

TABLE III.

*Periodic Classification of the Elements.*

| | Group 0 | Group 1 | Group 2 | Group 3 | Group 4 | Group 5 | Group 6 | Group 7 | Group 8 |
|---|---|---|---|---|---|---|---|---|---|
| 1 | | H 1 | | | | | | | |
| 2 | He 4 | Li 7 | Gl 9.1 | B 11 | C 12 | N 14 | O 16 | F 19 | |
| 3 | Ne 20 | Na 23 | Mg 24.4 | Al 27.1 | Si 28.3 | P 31 | S 32 | Cl 35.5 | Fe 55.9 Co 59 Ni 58.7 |
| 4 | A 39.9 | K 39.1 | Ca 40.1 | Sc 44.1 | Ti 48.1 | V 51 | Cr 52.1 | Mn 55 | |
| 5 | .. | Cu 63.6 | Zn 65.4 | Ga 70 | Ge 72.5 | As 75 | Se 79.2 | Br. 79.9 | Ru 101.7 Rh 103 Pd 106.5 |
| 6 | Kr 82.9 | Rb 85.5 | Sr 87.6 | Yt 88.7 | Zr 90.6 | Cb 93.5 | Mo 96 | .. | |
| 7 | .. | Ag 107.9 | Cd 112.4 | In 115 | Sn 118.7 | Sb 120.2 | Te 127.5 | I 126.9 | |
| 8 | Xe 130.2 | Cs 132.9 | Ba 137.4 | La 138.9 | Ce 140.3 | .. | .. | .. | |
| 9 | .. | .. | .. | .. | .. | .. | .. | .. | Os 191 Ir 193.1 Pt 195.2 |
| 10 | .. | .. | .. | Rare Earth Metals. | .. | Ta 181 | W 184 | .. | |
| 11 | .. | Au 197.2 | Hg 200.6 | Tl 204 | Pb 207.2 | Bi 208 | .. | .. | |
| 12 | .. | .. | Ra 226 | .. | Th 232.5 | .. | U 238.2 | .. | |

of the relation of the occurrence of the elements to their position in the periodic table, shown by the study of minerals and of igneous rocks, and taking into consideration the chemical relations of the various elements.

### THE PETROGENIC AND METALLOGENIC ELEMENTS.

In Table III is presented the periodic classification of the elements, as usually given.[17] The atomic weights are stated in round numbers, most of the elements of the "rare earths" are omitted, as their relative positions are still in dispute, as are also the radio-active elements, except radium.

Here, as has been commonly recognized, the most abundant elements in the earth's crust, being of generally low atomic weight (or atomic number), occupy the upper part of the scheme, forming the series 1 to 4 of Groups 1 to 8. These, with some others in series 6 and 8 to be mentioned presently, may be called the "Rock Elements," as they are the essential elements, in greater or less amount, of the igneous rocks of the earth's crust, of which they constitute at least 99.9 per cent. by weight.

In the lower part of the scheme are elements of higher atomic weight which, with others, in series 5 and 7, to be mentioned later, are but seldom, if ever, found in determinable quantities in igneous rocks, but which occur chiefly as ores, or as native metals. These may therefore be called the "Ore Elements." There would seem to be a very definite and distinctive difference between these two groups as regards their general chemical relations—a difference that has apparently not heretofore been observed.

Intermediate between the upper and lower part of the scheme is a zone, series 5 to 8, including elements that our study of minerals and rocks shows to belong partly to the rock elements, and partly to the ore elements. It is found that their relations to the one or the other is clearly distinguished by tracing a meander that separates them into alternate or interlocking vertical columns, the spaces thus made opening above into the division of the rock elements and below into that of the ore elements. Thus, as we shall see, Rb and Cs, Sr and Ba, Yt and La, Zr and Ce, Cb?, and Mo are to be considered as rock elements; while Cu and Ag, Zn and Cd, Ga and In, Ge and Sn, As and Sb, S, Se, and Te, and

[17] This table is based on that given by Clarke, *U. S. Geol. Survey Bull.* 695, p. 37.

Br and I, are ore elements. These differences are indicated by some of their general chemical relations and by the facts of their occurrence as minerals, that is, as components of the earth's crust.

Flanking the main part of the table is Group 0, the column of the inert gases, from helium to xenon. At the bottom are the radio-active elements, chiefly radium, thorium, and uranium, with others (some more or less hypothetical) that have been recently discovered. On the right is the column of Group 8, that of the triads. Of these, iron, cobalt, and nickel, are to be considered as rock elements, and the two triads of the platinum metals as ore elements.

It may be as well to suggest here, and to use henceforward, two terms as a matter of convenience. We may call the "rock elements" *petrogenic* and the "ore elements" *metallogenic*. These terms are not only short and self-explanatory but, having an adjectival form, are convenient for use. The distinctive chemical differences between the petrogenic and the metallogenic groups of elements, as regards their *occurrence in the earth's crust*, will be now set forth; and, it may be said, that these differences seem to divide them into two "natural" groups, which may be of significance in a study of the constitution of the earth. In the present paper it is best not to go very deeply into technical mineralogical details, and only the main facts will be stated, leaving the details for presentation elsewhere.

The petrogenic elements occur normally in nature as primary minerals, forming oxides, silicates, fluorides, and chlorides; but never, or only exceptionally, as sulphides, selenides, tellurides, arsenides, antimonides, bromides, or iodides. With the exceptions of iron and nickel, they are never found in the form of native metals. The metallogenic elements, on the other hand, normally (as primary minerals), form sulphides, selenides, tellurides, arsenides, antimonides, bromides, or iodides, but only seldom and exceptionally do these occur as (primary) silicates, oxides, fluorides, or chlorides. They are frequently met with as the "native" elements. There are, it is true, some exceptions to these statements (as with iron, which forms three common sulphides, and with tin which occurs mostly as the oxide); but taken broadly, and as applying to the two several groups as a whole, the distinction seems to be valid.

The oxides of many of the electropositive petrogenic ele-

ments are known to occur as minerals; those, that is, that are stable and are not readily soluble. They include periclase ($MgO$), corundum ($Al_2O_3$), quartz and tridymite ($SiO_2$), rutile ($TiO_2$), ilmenite ($(Fe,Ti)_2O_3$), chromite ($FeO.Cr_2O_3$), pyrolusite ($MnO_2$) and other oxides of manganese, hematite ($Fe_2O_3$), and magnetite ($Fe_3O_4$). All of the electropositive petrogenic elements form silicates, and, indeed, they form the overwhelming majority, certainly 99.9 per cent. of all known silicates by weight. Besides the simple silicates are borosilicates, fluosilicates, titanosilicates, and zirconosilicates, all of them salts of petrogenic elements. A few sulphosilicates are known, but they are very rare, and there are no known arseno-, antimono-, seleno- or tellurosilicates.

Fluorides and chlorides of sodium, potassium, ammonium, magnesium, calcium, aluminum, cerium, iron, and manganese, are known as minerals, and some of them are quite common, as NaCl, KCl, and $CaF_2$. On the other hand, neither bromides nor iodides of these elements occur as minerals, though there is an excessively rare calcium iodate. Fluorine replaces hydroxyl in several silicates, as in topaz and chondrodite, and it is also present in small amounts in hornblendes and micas, while chlorine is present in small amount in some silicate minerals, as those of the sodalite and scapolite groups. Until we reach vanadium, with atomic weight 51 and atomic number 25, no sulphides occur as minerals, except calcium sulphide, which occurs as a rare mineral (oldhamite) but only in a few meteorites. A very rare vanadium sulphide, found only in one locality, an extremely rare chromium-iron sulphide, occurring only in a few meteorites, and a rare terrestrial manganese sulphide are known. No arsenides, selenides, or tellurides of these elements, or of those preceding them in atomic number, are known. With the iron group, we find sulphides very common, the sulphides of iron, pyrite, marcasite, and pyrrhotite, being common minerals, and sulphides of nickel and of iron and nickel, as well as their arsenides, are widespread ore minerals. Sulphides and arsenides of cobalt are also fairly common. The sulphide of molybdenum is the only usual mineral of this element, though a few other minerals containing it (as secondary molybdates) occur. Selenides, tellurides, and antimonides of iron are apparently unknown in nature, though of nickel there are some very rare minerals of this character. It will be seen that such

compounds (sulphides, arsenides, etc.) of the petrogenic elements are all of those of rather high atomic weight and in the groups of highest valence, especially common in the triad group iron-cobalt-nickel.

Turning to the metallogenic elements, we find that many of them exist in nature uncombined, notably copper, silver, gold, mercury, arsenic, antimony, bismuth, sulphur, selenium, tellurium, and the metals of the platinum group. Native zinc, lead, tin and tantalum are also reported, but in some cases doubtfully.

As minerals the oxides of these elements either do not exist (as of gold, mercury, and the platinum metals), are of extreme rarity, or are certainly or almost certainly of secondary origin, as those of copper, zinc, arsenic, and antimony. Tin oxide, the common ore of this metal (cassiterite), is an apparent exception, but it would seem to be possible that, in some cases at least, it is of secondary origin, a sulphide being the primary compound.

Primary silicates of the metallogenic metals are very rare. There are none of gold, silver, mercury, thallium, tantalum, tungsten, or the platinum metals. Silicates of copper and zinc are quite common, but are in all cases almost undoubtedly of secondary origin. There are, however, silicates (possibly primary) of tin, lead, and bismuth, but they are mineral rarities, and many mineralogical museums and collections have no specimens of them.

No fluorides of any of the metallogenic elements are known as minerals, but insoluble chlorides and oxychlorides of copper, silver, mercury, and lead are known, though rare. On the other hand, as native bromides and iodides we know only those of copper, silver, mercury, and lead—all metallogenic elements.

The typical, and by far the most abundant, native compounds of all these metallogenic elements then are the sulphides, arsenides, antimonides, selenides, and tellurides, with the complex sulphosalts. These form the main, and in some cases the only, sources of most of the metals. Indeed, of gold, mercury (except the common sulphide, a chloride, and two doubtful iodides), and thallium (except a rare sulphide), the only native compounds known are selenides and tellurides; and conversely, the only native selenides and tellurides known are of copper (rare), silver, gold, mercury, thallium, lead, and bismuth, except that there is a very rare nickel telluride. Oddly enough the only native compounds known of

the platinum metals are ruthenium sulphide and platinum arsenide, no selenides or tellurides of these being known.

Returning to the intermediate interlocking meander zone, it may be well to point out some features that show to which of the two main groups the several elements there belong, and allude to another feature of interest regarding this part of the table.

Rubidium and cæsium are only known as silicates, cæsium forming the rare polysilicate pollucite, and both entering in small amount into other silicates, as beryl, lepidolite, and a few others. Strontium and barium, apart from their sulphates and carbonates of secondary origin, enter only into silicates, a barium silicate forming a member of the feldspar group, and both being the bases in some of the hydrous zeolites. The proper position of yttrium and lanthanum, in Group 3, is somewhat uncertain, but they both enter into the composition of various silicate minerals, and are not known as sulphides, arsenides, etc. The position of zirconium and cerium is quite clear; both form silicates, zircon being especially widespread among granitic rocks, and they also enter into the composition of some members of the pyroxene group. The position of columbium (niobium) is also somewhat uncertain, as no silicates of it are known; but it is the base in some titanates, and its general affinities as to mineral occurrence would place it almost surely with the petrogenic elements. Closely related to it, and occurring with it almost always, is tantalum, whose true place is uncertain. Both these elements, however, are very rarely met with. The researches of Hillebrand [18] have shown that molybdenum is very widely distributed among the more silicic igneous rocks, such as granites, so that, even though its most abundant mineral is the sulphide, it should be reckoned with the petrogenic elements.

Of the intermediate metallogenic elements, the positions of copper and silver are unquestionable, as both occur combined most frequently as sulphides, and other such minerals. Silver does not occur as a silicate or oxide; but silicates and oxides of copper are not uncommon, though these are of secondary origin. The same may be said of zinc and cadmium, the oxide and silicate of zinc being secondary. Gallium and indium are found only in zinc sulphide (sphalerite), and germanium occurs only as a sulphide with silver and tin. Though tin is most commonly met with as the oxide (as

[18] Hillebrand, W. F.: *Amer. Jour. Sci.*, vi, p. 209, 1898.

well as a rare silicate), yet sulphides of it are known, so that, in spite of its frequent occurrence as oxide, it is to be reckoned with the metallogenic elements. Arsenic and antimony, as well as selenium and tellurium, belong, of course, in this group; as does sulphur, the necessary inclusion of which among the metallogenic elements carries them somewhat into petrogenic territory, and renders the meander somewhat unsymmetrical toward this end. Bromine and iodine, as we have seen, are only met with in nature in combination with metallogenic elements, so that they may probably be placed with these. The metals of the ruthenium and the platinum groups clearly belong here, because of their occurrence as metals, and because of the existence of the sulphide of ruthenium and the arsenide of platinum as the only native compounds known.

On referring to the periodic classification presented in Table III, it will be seen that the intermediate, meandered zone, where the petrogenic and the metallogenic elements interlock, shows a very large proportion of elements with atomic weights that are quite far removed from whole numbers, which would imply, as has been suggested by Harkins, that this is especially the region of isotopes. Whether this is fortuitous, or whether it is (if it be true) connected with the division here suggested of the elements into the petrogenic and the metallogenic groups, is quite unknown, and it is needless here to speculate upon the subject.

It should be mentioned that the relations between the positive and the negative elements, and their occurrence in nature as minerals, as set forth above, form an elaboration and an extension of what Clarke has already called attention to,[19] namely: "In combination unlike elements seek each other, and yet there appears to be a preference for neighbors rather than for substances that are more remote. . . . The elements of high atomic weight appear to seek one another, a tendency which is indicated in many directions, even though it cannot be stated in the form of a precise law. The general rule is evident, but its significance is not so clear." A possible significance, or rather a possible connection between this rule and the occurrence of the elements, both as to their relative abundance and their mutual relations, in the earth's crust and below it, may be suggested here, as a somewhat speculative hypothesis.

[19] Clarke, F. W.: "The Data of Geochemistry," *U. S. Geol. Survey, Bull.* 330, p. 35, 1908; and *Bull.* 695, p. 39, 1920.

### THE INTERIOR OF THE EARTH.

The hypothesis (already adverted to), that the interior of the earth is composed, at least in part, of an iron-nickel alloy like that which composes many meteorites, is commonly held. This is based on the mean density of the earth, its rigidity and magnetic character, and the composition of many meteorites, the siderolites, which may be regarded as fragments of a preëxisting large body. Following Charles Darwin and Durocher, who published their views in the first half of the last century, the idea is now held by many that the material composing the interior of the earth is arranged, in a general way, according to relative density,[20] there being a nucleus or core of iron-nickel and possibly other heavy metals, above this a zone of heavy silicate rocks, and at the surface the lighter silicate rocks of the "crust," but presumably passing gradually one into the other, without sharp borders.

Wiechert and Knott have recently shown, through a study of the propagation of earthquake waves, that there is a change in the material, or in the physical properties of the material, at a depth of about 0.5 of the earth's radius. Still more recently, by laboratory measurements of the compressibility of rocks, as well as by the study of earthquake waves, Adams and Williamson,[21] of the Carnegie Geophysical Laboratory, have shown that the much greater density of the interior of the earth cannot be accounted for by the compressibility of the materials, whether rocks or metals. They are also led to the conclusion that, while there is segregation of heavier material toward the centre, that the change is continuous, and not discontinuous, as is held by Wiechert and Knott.

Following the views of Adams and Williamson, and accepting a lower zone of nickel-iron beneath the silicate "crust," I would suggest here the idea that the central core, the real nucleus, of the earth is composed of the metallogenic elements, that is, the elements or metals of highest atomic weight, either as "native" metals, or possibly in the form of selenides, tellurides, arsenides, antimonides, bromides, and iodides. Above this would be the nickel-iron zone, and above this the silicate crust.

[20] See, for example, Suess, "The Face of the Earth" (English translation), Vol. iv, p. 547, 1909; Daly, "Igneous Rocks and Their Origin," pp. 162–168, 1914.

[21] I must express my thanks to my colleagues for permission to mention briefly here some of their conclusions, which have not yet been published.

We cannot here discuss this suggestion in all its rather complex aspects. But the somewhat intermediate chemical character of the metals of the iron group, with manganese and chromium, is in accord with the hypothesis, differing as they do from the other petrogenic elements in their occurrence as sulphides and arsenides, in which they resemble the heavier metallogenic elements. Iron is the fourth most abundant element, and if the position of the nickel-iron zone, or a zone of mixed alloy with silicate rocks, were comparatively near the surface, this would be expected. The occurrence of iron-bearing basalts at the surface (met with in Greenland, Russia, Spain, and elsewhere) is also in line with this supposition.

Again, as on this supposition the true metallogenic elements are most deeply buried, their relative scarcity at the surface is readily understandable. Forming the nuclear core, not only would their total volume be relatively small, but it would also be difficult for them to find their way, even as vaporized or soluble compounds, from the great depths to the surface. The generally low melting-points of the ore minerals is also in line with the opinion of Adams and Williamson that the deepest interior is not entirely a rigid solid, but more in the nature of a very viscous, thick liquid, which damps the transverse earthquake vibrations. The possible factor of the disintegration of the elements of highest atomic weight must be taken into account, but more cannot be said here on this topic.

It is of interest to note that this idea, that the elements of higher atomic weights, the metallogenic elements, occupy for the most part the deepest portions of the earth's interior, is in harmony with Abbot's view as to the distribution of the elements in the sun.[22] He points out that the elements showing the most intense spectrum lines are those of low atomic weight, with the exception of the negative elements, none of which (with the possible exception of oxygen), for some unknown reason, show solar spectral lines. It is interesting to compare Abbot's table of intensities (page 91) with the elements as presented in our Table III of the periodic arrangement. It will be seen that the first 22 elements showing the most intense lines are all terrestrially petrogenic elements, and that (apart from the negative elements) all the terrestrial petrogenic elements are among those that show

[22] Abbot, C. G.: "The Sun," 1911, pp. 91, 94, 99, 104, and 252 ff.

the more intense lines, with the curious exceptions of glucinum, cerium, and especially potassium, which show but very weak lines. The order is not the same, but the first ten elements in order of spectral intensity include calcium, iron, hydrogen, sodium, magnesium, silicon, aluminum, and titanium, which, with oxygen, potassium, and phosphorus, are the first eleven elements in order of abundance in the earth's crust. On the other hand, the metallogenic elements show the least intense or no solar spectrum lines. Thus in Abbot's intensity tables Nos. 23 to 36 (the last) include in order palladium, copper, zinc, cadmium, germanium, rhodium, silver, tin, and lead. The metals of the platinum group, with tungsten, bismuth, mercury, thallium, and one or two others, give extremely feeble or doubtful lines. As Abbot shows, taking the elements in groups of order of intensity, this diminishes with increase in the mean atomic weight of the group.

Abbot explains this distribution, to which the only real exceptions are cerium, glucinum, and potassium, by the supposition that "the explanation of the decrease of intensities with increasing atomic weights seems to depend on the depth of these gases below the sun's surface," and this supposition is confirmed by the spectrum observations of displacements of the lines of various elements due to pressure and those that show in the "flash" spectra during eclipses. The coincidence between the occurrence of the elements in the earth and in the sun, as regards relative abundance and depth, is apparently so very close and detailed as to be suggestive of a similar arrangement in both bodies. It is also quite in harmony with the general idea of arrangement according to specific gravity or "gravitative adjustment."

We may conclude therefore that the metallogenic elements are rare on the earth's surface and do not show intense spectrum lines in the sun, because they are too deeply buried in both. Connected with this, however, is the difference in the chemical relations already pointed out, the significance of which is as yet problematical.

It might be pointed out here that such a theory of the vertical distribution of the elements seems to be opposed to Chamberlin's hypothesis of the planetesimal origin of the earth, though the matter cannot be discussed in this paper. Attention may only be called to the fact, probably very significant in this connection, that the melting points of the oxides and silicates, the typical natural

compounds of the petrogenic elements, are much higher than those of the sulphides and arsenides, the typical natural compounds of the metallogenic elements. The bearing of this will be discussed elsewhere.

Much more might be said of this suggestion of the distribution of the elements of highest atomic weight and greatest density at the centre. The idea is not wholly new, having been held speculatively by others. One might even recall, to pass from science to fiction, that the idea was, in a way, foreshadowed by Jules Verne, who in one of his stories describes a comet or huge meteorite composed of telluride of gold.

### CORRELATION OF THE ELEMENTS.

But we have wandered far from our proper topic, the crust of the earth, having reached not only the centre of the earth, but the sun, and become enmeshed in somewhat transcendental chemical speculation. Let us come back to the surface of the earth.

Before returning, however, to the consideration of the actual crust and its rocks, it may be as well to examine briefly a feature of the mutual relations of the elements (for the most part petrogenic), that is shown us by chemical study of the rocks and of the many minerals with which we are acquainted. Since the chemical analysis of rocks and minerals began to assume large proportions, so that sufficient and sufficiently accurate data became available, it has been noticed that certain elements are prone to be found in rocks of certain general compositions, and also in association with one another in minerals. In other words, there has been observed a certain correlated distribution of the elements in the earth's crust, that is, in the rocks and minerals composing it, by which certain of the elements tend to occur together in greatest abundance or most often, while other elements are seldom if ever found along with these. As this is a matter of considerable interest and importance from the mining engineer's point of view, several attempts have been made to formulate the relations, and it will be pertinent to give a very brief account of the subject.

Among the earliest of the more modern workers to investigate this problem are Vogt, Kemp, and De Launay,[23] who confined

[23] Vogt, J. H. L.: *Zeits. Prakt. Geol.*, 1898, p. 326; Kemp, J. F.: "Ore Deposits," 3d edition, pp. 34–37, 1900; De Launay, L.: "La Science

their attention chiefly to whether the various elements considered were most abundant in the more or the less siliceous rocks.

The writer pointed out [24] that "the relations are more complex and are dependent, not so much on the relative amount of silica, as on the relative amounts of other constituents, notably soda, potash, iron, magnesia, or lime." Such relations of common association are shown, in part among the most abundant constituents of rocks and minerals, and in part among the rarer ones, generally in connection with the more abundant. For the most part, the relations so far observed, which may be considered as best established, are confined to the petrogenic elements, as would be expected, but there seem to be similar relations, not yet quite clear, between some of the metallogenic and the petrogenic elements.

Broadly speaking, silica, alumina, soda, and potash tend to go together; thus the rocks that are highest in silica have, in nearly all cases, alumina and the alkali metals as the next most abundant constituents. At the same time, the alkali metals, and lime (not iron or magnesia), tend to go with alumina; so that a very large number, and among these the most common, of the silicate minerals are silicates of alumina and (or aluminosilicates of) soda, potash, and lime. The iron oxides and magnesia do not show nearly so strong a tendency to combine with silica or with alumina. In this connection may be mentioned a tendency toward combination with (or affinity for) silica, which may be expressed thus:

$$K_2O > Na_2O > CaO > MgO > FeO.$$

That is, potash will endeavor to take all the silica that it can, so far as is compatible with certain physical conditions, soda next, and so on; iron being the only very abundant element (except silicon) that commonly forms an oxide alone, that is to say, uncombined with silica. This general law or rule, which is based on the most generally observed relations among rock-forming minerals, is the basis of a recently introduced classification of igneous rocks, and it

Géologique," p. 637, 1906. Cf. also Hillebrand, W. F.: *U. S. Geol. Survey, Bull.* 700, p. 25, 1919; and Clarke, F. W.: *U. S. Geol. Survey, Bull.* 695, p. 13, 1920.

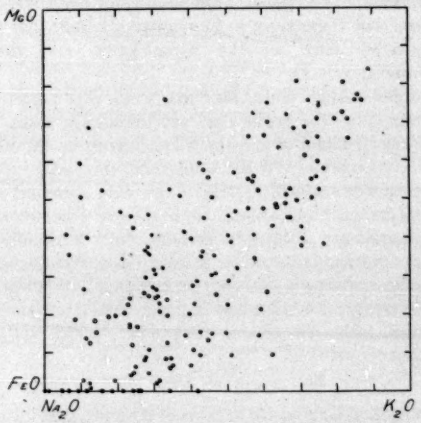
[24] Washington, H. S.: *Trans. Amer. Inst. Min. Eng.*, p. 751, 1908; Cf. Washington, "Manual of the Chemical Analysis of Rocks," 1st edition, p. 14, 1904; 3d edition, p. 17, 1919.

gives promise of fruitful application in the future. A similar "order of affinity" as regards alumina is also true of the same elements.

Magnesia and the iron oxides tend to go together, or to replace each other in many minerals, which seems to be of much the same import, and these oxides are, as we have seen, generally opposed to soda, potash, and lime.

Among the more interesting of such correlations are those of soda and iron on the one hand, and of potash and magnesia

FIG. 1.




Relation of Na and Fe to K and Mg.

on the other, these two pairs tending to go together. This is shown by many minerals, the details concerning which it is unnecessary to give here, though there may be mentioned the sodic pyroxenes, which contain much iron and little if any magnesia, and the potassic micas, which generally contain more magnesia than iron along with the potash. Study of many analyses of igneous rocks also brings this relation out very clearly, and it is expressed in the above figure (Fig. 1) published some years ago.[25] In this the abscissas represent the relative amounts

[25] Washington, H. S.: *Proc. Natl. Acad. Sci.*, i, p. 574, 1915.

of soda and potash, and the ordinates those of iron oxide and magnesia. The general drift of high soda coincident with high iron, and conversely of high potash with high magnesia (though such rocks are comparatively few), is clearly shown, and, as the data are derived from numerous analyses, and are substantiated by many others more recently made, the general "drift" may be considered as fairly well established. That the points fall in a rather broad zone, instead of along a narrow line, is to be attributed to the complications that may be introduced in such correlations by the presence of silica, lime, and possibly aluminum or titanium.

It may be mentioned here, *en passant*, that, curiously enough, the same correlation between these two pairs of elements, soda and iron, and potassium and magnesium, seems to hold good in the organic world.[26] This is apparently shown by the following facts: In autotrophic plant metabolism potash is an essential element, as is also magnesium, in that chlorophyll (which in the leaves acts as the carbon-transferring substance) is a magnesium salt of a complex organic acid, while sodium and iron are generally toxic toward (at least the higher, gymnospermous and angiospermous) plants. On the other hand, sodium, rather than potassium, is the alkali metal essential to the higher animals, salt being a very necessary article of diet (in part because of its chlorine, and in part because of its sodium, content), and sodium is present in the blood plasma; and at the same time, hæmoglobin and its derivatives (which act as oxygen carriers, and are analogous to chlorophyll in plants) are iron salts of organic acids closely related to that of chlorophyll; while, similarly potassium and magnesium are more toxic toward the higher animals than are the other pair.

Let us now pass briefly in review some of the correlations that are shown in igneous rocks by the rarer, and generally petrogenic, elements with the most abundant ones. In the first place, the rocks that are dominantly sodic seem to show the greatest tendency towards the segregation of many of the rarer elements. Thus, lithium, zirconium, cerium (and some of the other rare earth metals), chlorine and fluorine, and probably glucinum and tin, are found most often, both as components of minerals, and in rocks, that are high in soda. Barium seems to be most abundant

[26] Washington, H. S.: *Proc. Natl. Acad. Sci.*, ii, p. 623, 1918.

in those that are high in potash; titanium,[27] manganese, vanadium, nickel, and cobalt, in those that are specially high in iron; and chromium and platinum in those that are high in magnesium. Of the proclivities of the more truly metallogenic elements, as gold, silver, mercury, lead, and zinc, we know little as yet, but further study may indicate such relations, if they exist.

It is needless to enlarge here on the bearing of such observations on the practical search for ores and metals, especially those of the rarer kinds, some of which are now coming into prominence, such as tungsten and tantalum for electric lights, and zirconium for refractories. It will be self-evident that a knowledge of the rocks of a region can thus give us a clue as to what elements, or their ores, may be most likely met with, so that, for instance, we would not search for platinum in a region of sodic rocks, but would here rather look for the minerals of cerium, the rare earths, uranium, or tungsten.

### COMAGMATIC REGIONS.

Let us now return to the earth's crust and endeavor to answer the second question propounded above, namely, whether all large portions of the crust are alike in general, or whether they show marked differences; that is, whether the crust is essentially alike or unlike over different areas.

Nearly fifty years ago Vogelsang[28] pointed out that the igneous rocks of certain districts showed certain textural or mineral characters in common, which served to distinguished them from the rocks of other districts. The same idea was expressed later by Judd,[29] and still later by Iddings,[30] the latter showing that the differences between different districts were referable ultimately to differences in the chemical composition of the rocks. Such districts were called "geognostische Bezirke" by Vogelsgang, "petrographic provinces" by Judd, the latter name being that in common use, Iddings using the term "consanguinity," while the writer later[31] called them "comagmatic regions," to indicate the

[27] Titanium also evinces preference for sodium, like its congener zirconium.

[28] Vogelsang, H.: *Zeits. deutsch. geol. Ges.*, xxiv, p. 525, 1872.

[29] Judd, J. W.: *Quart. Jour. Geol. Soc.*, xlii, p. 54, 1886.

[30] Iddings, J. P.: *Bull. Phil. Soc. Wash.*, xii, p. 128, 1892.

[31] Washington, H. S.: *Carnegie Inst. Publ., No.* 57, p. 5, 1906.

idea that the various rocks of a given region are derived from a common magma, by processes of so-called differentiation. Into the discussion of differentiation we cannot even begin to enter here, though it forms one of the most important and most complex features of petrology, the science of rocks.

The proper study of petrographic provinces, or, as we shall here term them, comagmatic regions, is as yet, so to speak, in its infancy. Only a few regions have been described at all adequately from the most general point of view, such as the Christiania region in southern Norway, that of Central Montana, the Yellowstone Park, and the volcanoes of western Italy; and these descriptions leave much to be desired.

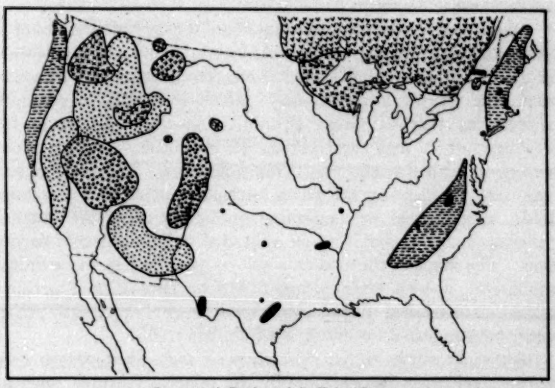
Indeed, even the fundamental data for our definition of a comagmatic region are somewhat uncertain and the application of the idea is somewhat loose. Thus, considering the time element, the life of a region may extend over many geological periods, as that of Great Britain from the Silurian to the Tertiary; or it may be confined to but little more than one period, as with the western Italian volcanoes. The areal extent may vary from many thousands of square miles to a few hundreds, though we are beginning to believe that the smaller "regions" are probably to be regarded as but parts of larger ones. The shape of the area may also vary; it may be more or less equilateral, a long zone, either broad or narrow and perhaps forked, or be evident only as small, separate, and apparently structurally unconnected occurrences of similar rocks.

Although some of the characters of any given region may be most evidently recognizable by the mineral features, such as the color of the pyroxenes or the peculiarities of the feldspars, yet these are all dependent on the prominent chemical characters of the magma, so that the chemical characters constitute the fundamental basis of distinction and characterization. In order to show the reader how, and in how far, the chemical characters of various portions of the earth's crust may differ, it will be well to note very briefly some of the best-known comagmatic regions of the earth, stating only their most prominent chemical features and omitting all details.

In the United States we find a long zone of disconnected areas whose rocks are dominantly sodic. This zone apparently begins in southwest Greenland, appears as a group of very similar small

areas in Ontario, Quebec, and New England (the so-called Novanglian region), appears in New Jersey, Virginia, probably North Carolina, in Arkansas, and finally as several areas in Texas. It is apparently continued south into Tamaulipas in Mexico; and what may be a continuation of it appears in some of the Antilles, in Brazil, and as far south as Paraguay. These areas in the United States are marked solid black in Fig. 2. The large "Canadian shield" around Hudson's Bay forms another region, which is dominantly calcic (anorthosites), marked with v's on the map. Along the Appalachian uplift, and probably extending into

FIG. 2.



Comagmatic Regions of the United States.

Maine, is another region, the rocks of which are characteristically rather sodic granites, though some very unusual rocks occur along this zone. This is marked with dashes (—) on the map. The sodic areas just mentioned may be connected with this. West of the Appalachians we find a few, small sporadic occurrences of peculiar rocks, high in potash and magnesia, as in New York, Pennsylvania, Kentucky, and Arkansas, which seem to be distinct from the preceding, and which may represent the great body of magma that underlies this part of the Mississippi Valley.

Around Lake Superior, in Minnesota, Wisconsin, and Michigan, and probably extending into Canada to the north, is an area

of igneous rocks that are low in silica but high in lime and iron oxides. To the last feature is due the importance of this region for its very abundant iron ores. It is marked with x's on the map. In the southern part of the Mississippi Valley, about the Ozark uplift, are some small and as yet little-studied occurrences of granitic rocks, which seem to form a distinct region.

West of the Mississippi Valley the comagmatic relations are more complex, as are the geological structures, but we can distinguish some fairly well-defined comagmatic regions. One of the most clearly marked is that which extends from, and possibly beyond, the Canadian border through central Montana, where it is represented by several volcanic centres described by Pirsson and others, into Wyoming, and with patches that probably represent it in eastern Colorado. These rocks are characterized by decidedly high alkalies, and with potash generally dominating soda. The areas are marked by x's on the map. Covering the great plateau of Colorado, Utah, and Nevada, with parts of Idaho and Wyoming (including the Yellowstone Park), and probably in northern New Mexico and Arizona, is a large and complex region, the rocks of which are decidedly of average composition, distinctly high in silica, moderate lime and alkalies, and low iron and magnesia. North, west, and south of this is a rather ill-defined region, whose rocks are similar but somewhat more calcic. The first is distinguished by small circles and the latter by dots on the map. These regions need further study, and it is doubtful if they should be treated separately.

In southern Idaho and in Washington and Oregon are the very extensive flow basalts of the Snake and Columbia Rivers, high in lime and iron oxides, which resemble chemically the rocks of the Lake Superior region and which are marked similarly on the map. The true relationship of these to the surrounding regions is doubtful. Along the Pacific Coast, chiefly in California, but extending to the north about as far as Puget Sound, there are indications of a narrow zone of decidedly sodic but rather highly silicic rocks. This may extend south along the west coast of Mexico, and may there be connected with the origin of the jadeite objects found in that country, the exact provenance of which is unknown. Thus may petrology aid archæology.

The long chain of the Andean volcanoes seems to form a continuation of the main Cordilleran region, which is continued

northward along the Aleutian volcanoes, and thence southward, along the west coast of the Pacific, through the volcanoes of Kamchatka, Japan, and the Philippines, and so on to the Dutch East Indies. This so-called "Circle of Fire" surrounds a large area, that of the Pacific Islands, whose rocks are dominantly basaltic, that is, low in silica and alkalies, and high in lime, magnesia, and iron, associated here and there with occurrences of alkalic rocks.

In Europe the various comagmatic regions are so numerous, so complex, and so little known from this point of view, that only a few need be mentioned. There is the extensive, though broken-up, region that embraces the British Islands and their outliers, with Iceland, Eastern Greenland, Jan Mayen, Spitzbergen and Nova Zembla, the rocks of which are dominantly basaltic. The highly sodic Christiania region in southern Norway has been well studied by Brögger, as has the calcic Bergen region by Vogt. Germany and Austria are filled with a complex of different regions, the relations of which are not yet clear, but which seem to be either dominantly sodic, as that of Bohemia, or with basaltic tendencies. The Alps and the Tyrol form a central region of prevailingly granitic rocks which differ markedly from the various and different regions that surround them; this is a point to which we shall recur. In Italy is the so-called Roman comagmatic region, embracing the volcanoes along the west coast from Bolsena to Vesuvius, the rocks of which are decidedly unusual in their very high potash, with considerable lime. A zone of distinctly sodic rocks appears to extend from southern France and eastern Spain, down Corsica and Sardinia, through the island of Pantelleria, into Tripoli. Hence, by way of Kordofan, this region is possibly connected with the highly sodic one that stretches from Abyssinia down the Ethiopian Rift Valley in East Africa, and which branches northwardly along the Red Sea and Arabia as far as Syria. At the east end of the Mediterranean, on the other hand, is a region embracing Greece and the Balkan Peninsula, the Archipelago, and western Asia Minor, whose rocks resemble very closely those of the Colorado plateau and of the Andes volcanoes.

We could go on thus over the surface of the earth, so far as its rocks are sufficiently well known chemically. Unfortunately, this is not the case with many large, and otherwise thoroughly studied, areas or regions, such as, for instance, the Greater An-

tilles, Cuba, Jamaica, Haiti, and Porto Rico. But this very rapid sketch will serve to give the reader some idea of how diversified, chemically, are the different portions of the earth's surface.

It has been suggested by several prominent petrologists [32] that the comagmatic regions may be referred genetically to two large types of magma or "provinces," called "alkaline" and "subalkaline" by Iddings, or "Atlantic" and "Pacific" by Becke. The latter goes so far as to attempt to ascribe all the comagmatic regions to two areas, the one dominantly alkalic and surrounding the Atlantic Ocean, and the other more calcic and surrounding the Pacific. Harker, furthermore, would connect these two main types of comagmatic region with two main types of crustal movement or stress, such as are recognized by Suess, which give rise to different types of coast, mountain formation, etc. In the opinion of the writer such recognition of but two types is not consonant with what we know of the general distribution of the igneous rocks. The whole subject is very complex, far too much so for proper discussion here, and the data available seem to the writer to be inadequate for very broad generalizations at present.

### CHEMICAL COMPOSITION AND ROCK DENSITIES.

The above outline of comagmatic regions leads us to the consideration of two subjects with which we may close this sketch of the chemistry of the earth's crust; that is, the relation between the chemical composition of rocks and their density, and that between these and the theory of isostasy.

In the preceding pages we have considered igneous rocks almost only from the chemical point of view. As we know, however, they are actually aggregates of definite chemical compounds, minerals, mostly silicates. Furthermore, we know that magmas of the same general chemical composition may crystallize as diverse aggregates of different minerals, according to the conditions that obtain during solidification. If we know the chemical compositions of the various rock-forming minerals, the quantitative mineral composition may be readily calculated from the chemical analysis of the rock. But from what has just been said, it is evident that the particular mineral aggregate to be

[32] For some general discussion of this and related topics, the reader is referred to: Harker, "The Natural History of Igneous Rocks"; Iddings, "Igneous Rocks," Vol. i; and Daly, "Igneous Rocks and Their Origin."

calculated will depend on the conditions controlling solidification. It is also obvious that, if we know the mineral composition and the densities (specific gravities) of the minerals, that of the rock as a whole may be readily calculated.

In the conception and elaboration of a system of classification of igneous rocks that was proposed some years ago by some American petrologists, the chemical composition of igneous rocks was regarded as their most fundamental character, and therefore that on which their classification was primarily based. But, in order to recognize the fact that they are actually mineral aggregates and so as to be able to compare them one with another on this basis, in spite of the various possibilities as to mineral composition introduced by the varying conditions of solidification, the chemical composition shown by analysis is calculated in terms of mineral composition according to one, uniform system, that is, one general assumption as to the minerals that are formed, or may be formed, from the particular magma. In this way, all igneous rocks are comparable and classifiable *inter se*, both chemically and mineralogically. The details of the procedure and the results of this system of classification cannot be gone into here, but may be looked for elsewhere.[33] It will suffice here to say that the general principles which are considered basal are the so-called "affinities" of the various basic oxides for, first silica, and second alumina, which have been given on a previous page, and which are deduced from the general knowledge of rock minerals. Carried out along the lines so laid down the results of the calculation from the data of the chemical analysis give a mineral composition which, although ideal, corresponds with the actual mineral composition in the great majority of cases.

Some years ago Iddings[34] pointed out that the density (specific gravity) of a rock as calculated from the calculated mineral composition on the assumption that the rock is holocrystalline, corresponds very closely with the actual density. This fact is of great interest; partly because of its justification of the funda-

[33] Cross, Iddings, Pirsson, and Washington: "A Quantitative Classification of Igneous Rocks," Chicago, 1903; Washington, H. S.: *U. S. Geol. Survey, Prof. Paper No.* 99, 1917. There is a considerable literature on this and other systems of the classification of rocks.

[34] Iddings, J. P.: "The Problem of Volcanism," p. 123, 1914. For a later and more detailed statement, see Iddings, *Amer. Jour. Sci.* (4), xlix, p. 363, 1920.

mental basis of the classification, and also because it thus furnishes a uniform means of comparing the densities, not only of particular rocks, but of the average rocks of different regions, and quite irrespective of such factors as those due to porosity or the presence of glass. Following the suggestion of Iddings, I have calculated the average densities of the continents, the ocean floors (represented by the lavas of the volcanic islands in the Pacific and the Atlantic), and of the igneous rocks of various countries and comagmatic regions, whose average chemical com-

TABLE IV.

*Average Compositions of Continents and Ocean Floors.*

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| $SiO_2$ | 59.09 | 60.01 | 61.09 | 59.66 | 61.77 | 58.30 | 59.99 | 53.56 | 50.63 | 50.06 |
| $Al_2O_3$ | 15.35 | 15.71 | 15.13 | 15.07 | 15.45 | 15.31 | 14.69 | 16.79 | 15.82 | 15.51 |
| $Fe_2O_3$ | 3.08 | 2.87 | 3.02 | 3.16 | 3.16 | 3.52 | 2.59 | 4.00 | 4.44 | 3.88 |
| FeO | 3.80 | 3.66 | 3.29 | 3.66 | 2.75 | 3.74 | 4.40 | 5.33 | 5.79 | 6.23 |
| MgO | 3.49 | 3.15 | 3.46 | 3.60 | 2.63 | 3.51 | 3.75 | 4.66 | 5.79 | 6.62 |
| CaO | 5.08 | 4.79 | 4.86 | 4.96 | 4.49 | 5.10 | 5.02 | 7.57 | 7.36 | 7.99 |
| $Na_2O$ | 3.84 | 3.89 | 4.07 | 3.72 | 4.09 | 4.84 | 3.49 | 3.57 | 4.27 | 4.00 |
| $K_2O$ | 3.13 | 3.06 | 2.68 | 3.39 | 3.22 | 3.28 | 3.02 | 2.32 | 2.31 | 2.10 |
| $H_2O$ | 1.14 | 1.01 | 1.04 | 1.24 | 1.23 | 1.26 | 1.19 | 0.93 | 1.47 | 1.16 |
| $TiO_2$ | 1.05 | 1.01 | 0.56 | 0.83 | 0.68 | 0.84 | 1.01 | 0.87 | 1.63 | 1.96 |
| $P_2O_5$ | 0.30 | 0.26 | 0.11 | 0.23 | 0.12 | 0.20 | 0.26 | 0.17 | 0.43 | 0.25 |
| MnO | 0.12 | 0.10 | 0.12 | 0.08 | 0.10 | 0.07 | 0.15 | 0.03 | 0.04 | 0.15 |
| Incl | 0.48 | 0.19 | 0.16 | 0.14 | 0.04 | 0.17 | 0.13 | .... | 0.07 | 0.08 |
| | 100.00 | 99.71 | 99.59 | 99.70 | 99.75 | 100.14 | 99.70 | 99.80 | 100.05 | 100.00 |

1. Earth. (5179 anals). Dens.=2.77. Elev. = +2252 feet.
2. North America. (1709 anals). Dens. =2.75. Elev. =1888 feet.
3. South America. (138 anals). Dens. =2.72. Elev. =2078 feet.
4. Europe. (1985 anals). Dens. =2.75. Elev. =930 feet.
5. Asia. (114 anals). Dens. =2.72. Elev. =3189 feet.
6. Africa. (223 anals). Dens. =2.77. Elev. =2021 feet.
7. Australia. (287 anals). Dens. =2.79. Elev. =805 feet.
8. Antarctica. (103 anals). Dens. =2.79. Elev. =? feet.
9. Atlantic floor. (56 anals). Dens. =2.85. Depth = −13,500 feet.
10. Pacific floor. (72 anals). Dens. =2.89. Depth = −14,820 feet.

positions were calculated by Doctor Clarke from the data in Professional Paper 99.

Before we discuss the densities it will be well to examine the average chemical compositions of the different continents and ocean floors, the data for which are given in Table IV. It will be seen that they vary considerably the one from the other, as well as from the general average of the earth's crust. Taking, for example, silica, the most abundant constituent, its percentages for North and South America, and especially for Asia, are decidedly above that of the earth's crust as a whole, while those

for Europe and Australia are only slightly above this. On the other hand, the silica percentages for Africa and Antarctica, and still more for the Atlantic and the Pacific floors, are very notably lower.[35] There is little difference, comparatively, in the figures for alumina, the alkalies, and the minor constituents, but those for the iron oxides, magnesia, and lime are distinctly lower in those cases where silica is higher and higher where this is lower.

The continental and oceanic averages shown above represent, in fact, different comagmatic regions on a large scale but in concise form. Though the data for some of them, as Asia, South America, Africa, and Antarctica, are not numerous enough to be wholly satisfactory, yet there would seem to be no valid reason for doubting that, taken as representing broadly the general chemical compositions of the larger structural divisions of the earth's surface, they may safely be assumed to give us a fairly trustworthy idea of the relations between them. In any case, they are the only large body of data that we have available, so let us use them provisionally and see to what results their consideration may lead.

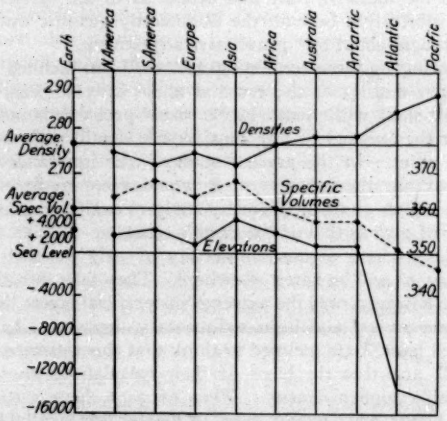
Before doing this, however, it will be as well to devote a few words to the average density of the crust as a whole, as this is an important factor in the consideration of isostasy, to be taken up later. It will be seen from the table that the average density of the crust is calculated, from the average chemical composition, to be 2.77. An average might be arrived at by considering all the determinations of specific gravity of rock specimens that have been made by the ordinary physical methods, and that are found abundantly in the literature. An average thus arrived at would seem to suffer from several disturbing factors that are eliminated by the method based on the chemical averages. Thus, it would include the densities of many lavas that are more or less glassy, which are decidedly lighter than holocrystalline rocks, and which, furthermore, are surficial rocks, not found at any considerable depth beneath the surface. It would also be seriously affected by the porosity of the surface rock specimens; and at great depths this must be very largely, or wholly, done away with by the pressure of the superincumbent crust, as shown by Van Hise and others. On the other hand, however, the density determina-

[35] If the analyses of the rarer rocks are disregarded, the density of the Atlantic floor is about 3.05 and that of the Pacific is about 3.10.

tions are probably more equably distributed among the various kinds of rocks than are the chemical analyses, which may reasonably be expected to include a possibly undue proportion of "interesting" and rarer types of rocks, as has been mentioned.

It is impossible at present to evaluate the relative influences of these several factors, but I might incidentally express my surprise that such a simple means of arriving at an estimate of the average density of the rocks of the earth's crust as is here suggested does not seem yet to have been attempted—at least nothing

FIG. 3.




Elevations and densities of the continents.

seems to have been published on the subject. An estimation that I am now making along this line is not yet complete enough for publication in this paper, but will be given later elsewhere.

On the whole, after due consideration of the several factors involved, I am inclined to put much greater weight on the final result arrived at from the averages of the chemical compositions. This, also, is subject to certain possible corrections in the future. It would seem to be probable that it is somewhat too high, as it does not include any, or at least a proportionate, number of analyses of many large areas which are almost certainly generally

granitic, and therefore relatively light. This applies to the interiors of Asia, South America, Australia, and probably Africa, to mention the larger divisions, and also to smaller ones, such as Spain, Egypt, South Africa, the Greater Antilles, and others. It is impossible now to estimate the magnitude of this correction.

On the other hand, if we are dealing with the rocks of the crust to any (humanly) considerable depth, such as the ten miles assumed by Clarke, and which might justly be placed at twenty or more, we meet with the possibility of a correction in the other direction, that is, toward a higher density. This conclusion is based on the ideas of Daly and others as to the existence of a basaltic substratum beneath the dominantly granitic outer shell, that is brought about by "gravitative adjustment."

Balancing up these conflicting factors, I am inclined to place the average density of the crust at about 2.75, at least for the uppermost shell, while that of 2.80 would probably be nearer the truth for the average of any considerable depth, such as twenty or more miles. In the present state of our ignorance and the paucity of our data, however, it would seem to be wisest to accept the figure given by the many analyses available, and assume a density of 2.77 as that of the earth's crust.

Geodesists have assumed a density of 2.67 for their studies of isostasy, as will be noted elsewhere. They take into their calculations, however, only the extremely superficial layers, including such strata as soil and light sedimentary deposits. As will be mentioned later, I am inclined to think that this estimate is much too small, and that the basis of their calculations should be a considerably higher density. That of 2.77, here assumed, or possibly better 2.75, would seem to be the best available under the circumstances.

### ROCK DENSITIES AND ELEVATIONS.

With the analyses in Table IV are given the calculated densities and the average elevations of the continents and the depths of the ocean floors referred to sea level. The general relations are graphically expressed in Fig. 3. The lowest graph is that of elevations, the uppermost is that of densities, while the intermediate one is that of specific volumes, or reciprocals of the densities, which serves better to bring out the parallelism. The sequence of the continents and oceans is arbitrary.

It will be evident from the Table IV and from the graphs that there is a close relation between the average densities of the continental masses and of the ocean floors and their average elevations or depressions. They stand in inverse relation to each other; that is, the higher portions of the earth's crust are composed of the lighter rocks, and the lower portions of the heavier. When it is remembered that these relations are shown by a very considerable number of averages based on a very large number of trustworthy analyses (the largest so far available), from all parts of the earth, the correspondences are too striking to be explicable by an appeal to chance or coincidence. This is even more obvious when we come to consider the relations in greater detail, as we shall do presently.

In discussing this subject it must be kept in mind that we are dealing with the averages of large areas and of many analyses, so that small and local details are lost. Thus a number of volcanoes show flows of heavy basalt covering lower flows or inner cores of lighter rhyolite or andesite. Again, it is not uncommon to find sheets of heavy basalt capping plateaus or forming the summits of their mountain remnants. But such apparent contradictions to the general law shown above are but local and minor details, insignificant as compared with the immeasurably greater masses of which they form but topographic surface features.

The general relations between rock density and elevation are also, and possibly more strikingly, seen when they are presented in greater detail, as is done in Figs. 4 and 5.[36] These are based on the average densities calculated from the average chemical compositions of the rocks of different countries and regions, as determined by Doctor Clarke. These represent the general elevations and corresponding average densities along two zones around the earth, the one roughly between Lat. 40°–50° N. and the other between Lat. 10°–20° S. It is to be understood that the graphs are much generalized, representing average densities and elevations, so that there is little detail.

The outer circle is that of sea level, and the irregular line that crosses it is a generalized graph of the land elevations and the ocean depths. Though the positions in longitude are approxi-

[36] For the suggestion of this method of presentation I am indebted to Dr. L. H. Adams, of the Geophysical Laboratory.

mately correct, the vertical scale of these is not that of the earth, as represented by the sea-level circle, but the heights and depths are very greatly exaggerated; otherwise the differences would not be perceptible in any practicable illustration. The portions of the land surface are, however, all drawn to the same (exaggerated) vertical scale, while that of the ocean depths is one-half of this. The elevations shown for the interiors of South America, Africa, and Australia are the continental elevation averages, as given by Murray.

The inner circle is that of the average specific volumes $\left(\frac{1}{2.77}=0.361\right)$, and the inner broken line, made up of arcs, is that of the average specific volumes of the portions of the earth's crust radially above the successive small arcs. Specific volumes (the reciprocals of the densities) are used instead of densities because the relations are brought out more clearly and immediately by the parallelism with the elevation graph shown by the former. The arc portions of the specific volume graphs in solid lines are the ascertained averages, while portions that are unknown, because of the absence of exposures of igneous rocks or for other reasons, are indicated by dotted arcs, their radial distance being roughly estimated, so far as is possible. These various arcs are connected by radial dotted lines.

The centre of the circles is the locus of the axis, seen from the North Pole, and is at the same time the zero point for the two graphs.

The graphs show the correspondence between elevation and specific volume so clearly that it is scarcely necessary to go into a detailed description; yet a brief summary of the northern zone may be of interest. This represents the conditions around a zone, which extends roughly between 40° and 50° North Latitude, varying somewhat north or south so as to include available data and complete the circle. The data on which the graphs are based are given in Table V.

Let us take a little journey around the earth along the northern zone (Fig. 4). Beginning at the Pacific coast the land gradually rises across California to the high plateau of Nevada and Utah, culminating in Colorado. Thence it slopes gradually down, across the Great Plains (Kansas) to the Mississippi Valley. Along this slope practically no igneous rocks are met with, except for sporadic and little-studied occurrences in the Ozark ridge. The slight rise

seen in the Mississippi basin is the level of the rocks of the Lake Superior region (Minnesota, Wisconsin, Michigan). East of this (Kentucky) few igneous rocks are known except those mentioned above, and the land slopes gradually up to the Appalachian

TABLE V.

*Average Densities, Specific Volumes, and Elevations.**

| | Density | Spec. Vol. | Average Elevation |
|---|---|---|---|
| | | | *Feet* |
| Earth | 2.77 | .361 | +2252 |
| North America | 2.75 | .364 | 1888 |
| United States | 2.75 | .364 | 2500 |
| South America | 2.72 | .368 | 2078 |
| Europe | 2.75 | .364 | 939 |
| Asia | 2.72 | .368 | 3189 |
| Africa | 2.77 | .361 | 2021 |
| Australia | 2.79 | .358 | 805 |
| Antarctica | 2.79 | .358 | ? |
| Atlantic floor | 2.85 | .351 | −12800 |
| Pacific floor | 2.89 | .346 | −15400 |
| Zone 40°–50° North Latitude. | | | |
| California | 2.742 | .365 | +3000 |
| Oregon and Washington | 2.773 | .361 | 2500 |
| Utah and Nevada | 2.717 | .368 | 6000 |
| Colorado | 2.735 | .366 | 7000 |
| Ozark Region | 2.728 | .367 | 2000 |
| Mich., Wisc. Minn. | 2.803 | .357 | 1000 |
| Appalachia (Penn. to Ga.) | 2.749 | .364 | 2000 |
| New England and N. Y. | 2.759 | .362 | 750 |
| Great Britain | 3.041 | .329 | 300 |
| France | 2.867 | .349 | 600 |
| Germany | 2.772 | .361 | 800 |
| Switzerland and Tyrol | 2.729 | .366 | 5000 |
| Austria-Hungary | 2.784 | .359 | 2000? |
| Urals and Caucasus | 2.829 | .353 | 2000+ |
| Pamirs | 2.72– | .368+ | 13000 |
| Japan | 2.723 | .367 | 1420 |
| Zone 10°–20° South Latitude. | | | |
| Andes | 2.717 | .368 | 6000? |
| East Brazil | 2.745 | .364 | 2500? |
| Africa (East and West) | 2.77 | .361 | 2021 |
| Madagascar and Reunion | 2.830 | .353 | 2199 |
| New Zealand | 2.749 | .364 | 2134 |

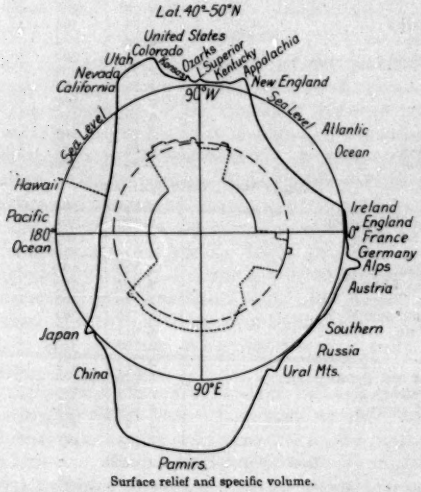
* For the hypsometric data I have consulted Murray, Scot. *Geog. Mag.*, iv, pp. 1-29, 1888; Gannett, *U. S. Geol. Survey, Ann. Rep.* 13, ii, d. pp. 283-289, 1892; Bull. No. 274, 1906; and other authorities.

ridge, and east of this, across New England in the graph, descends to sea level.

The floor of the North Atlantic is rendered very summarily, and the Azores and Iceland are about our only source of infor-

mation as to the composition of its floor. On the east coast of the Atlantic the British Isles rise to but a small height (on the average) above its surface. The average elevation of France is slightly higher; that of Germany (which is inserted in the zone a little out of latitude) is still somewhat higher, and thus we come to Switzerland and the Tyrol, the culminating portion of Europe. To the east of this, with an average elevation slightly greater than that of Germany, lies Austria-Hungary, and then to the east

FIG. 4.




Surface relief and specific volume.

the low-lying plains of South Russia. East of these are the Ural Mountains, and then (bending somewhat southerly) we pass through Turkestan and Persia, and reach the very high Pamirs, the "Roof of the World." Thence the surface slopes down across China, rises again in Japan, and again drops to the depths of the Pacific Ocean.

Let us now see how the rock densities, or rather the rock specific volumes, correspond with the elevations. This inner graph, it is to be remembered, represents the specific volumes,

that is the reciprocals of the densities, so that it is inverse to what the graph of the densities would be; that is, the heavier the average rock the nearer to the centre is it on this graph, and the lighter the farther away.

Starting with California, we find its specific volume arc above the average, and that of Nevada-Utah to the east still higher. The average of Colorado is a trifle lower, though the elevation is higher, and this is one of the very few notable exceptions to the general rule. We are ignorant of the igneous rocks beneath Kansas; they are indicated as but a little above the average, which is probably not very far wrong. There is a decided rise below the Ozark ridge (with its greater elevation), while the arc below the Mississippi Valley is represented by the small arc for the Lake Superior rocks, which are high in iron oxides and with high average density. Of the rocks below "Kentucky" (east of the Mississippi) we know little, but the Lake Superior arc is continued here because of the sporadic occurrences of some heavy peridotites in this region mentioned above. The graph rises sharply in the arc beneath the Appalachians, falling again beneath New England, which is distinctly below the average.

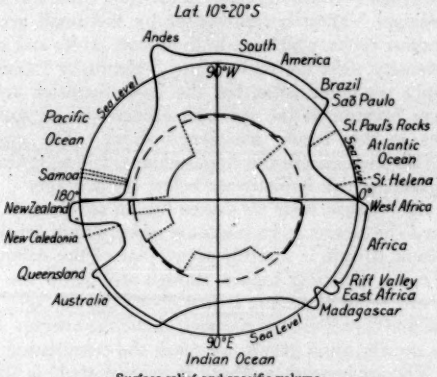
With the Atlantic floor we descend to an arc beneath it that is well toward the centre, as its rocks are of very high density. The arc for Great Britain is scarcely above that of the Atlantic floor, that of France distinctly higher, though still below the average, while the arc below Germany is just above the average.[37] With the Alpine and Tyrol arc we rise well above the average and here, just as in the elevation graph, we reach the culminating point of Europe. The arc beneath southern Russia (dotted) is placed at a level but slightly different from that of Great Britain because, though igneous rocks are rare in this district, there are occurrences in Volhynia of very heavy iron-bearing basalts. The arc beneath the Urals is but slightly above this, corresponding with the heavy rocks of these mountains which, it is to be remembered, are low and little more than large hills. Of the rocks of Persia and Turkestan we know but little, so the arc below this is dotted and slopes up to that beneath the Pamirs, or Central Asia. Here

[37] The specific volume arc for Germany should be but little above that of France to correspond with the relative elevations; it appears to be much higher because very many of the German analyses of the heavier rocks (diabases, basalts, etc.), the analysis of which is most liable to error, are of very poor quality, and are therefore omitted.

again our knowledge is far from precise, so that the arc is dotted and is placed at the level of that of the average of Asia, though it should probably be somewhat higher. Analyses of Chinese rocks are few, but they would seem to be in general like those of the Pamirs, though a trifle heavier. The Japan arc is above that of China, and east of this we reach the wide arc beneath the Pacific Ocean—the lowest of all, just as its rocks are the heaviest.

After the journey around the world that we have just made in the northern hemisphere it seems quite needless to make that in the southern (Fig. 5). The reader may follow the correspondence

FIG. 5.




Surface relief and specific volume.

for himself, remembering that the available data along this zone are far less numerous than along the northern; though we have good knowledge of the rocks of Madagascar,[38] Eastern Australia and New Zealand, and fair knowledge of those of the Andes and western Africa and the Ethiopian Rift Valley.

### ROCK DENSITIES AND ISOSTASY.

We come now to the final section of our study of the earth's crust, the application of the data just presented to an important

[38] The average density of Madagascar is unquestionably less than that here given, as is shown by the many more analyses now available. The

theory regarding the stability conditions of the crust, the theory of isostasy.

As far back as 1852 J. H. Pratt suggested that there was a deficiency of gravity beneath the Himalaya Mountains, basing this on the anomalous behavior of the plumb-bob.[39] He also propounded the view that the heavier portions of the earth's surface were sinking. Later, Dutton[40] first clearly expounded the idea that the various portions of the earth's surface, being laterally unlike or heterogeneous, are in a delicately balanced condition of equilibrium, so that the lighter portions (those of less specific gravity) tend to rise, and the heavier (those of greater specific gravity) tend to sink, the various portions thus balancing each other. This theory, later called isostasy (meaning equal standing), was taken up by Hayford, and he and William Bowie, of the U. S. Coast and Geodetic Survey, and others,[41] have done much to develop it, especially as regards its application to gravity problems. While still in dispute, especially as to some details, it is a well-recognized and generally accepted geodetic theory.

It will be well to quote, in part, Hayford's definition[42] of isostasy. Assuming a condition of lateral heterogeneity, he says: "Different portions of the same horizontal stratum may have somewhat different densities, and the actual surface of the earth will be a slight departure from the ellipsoid of revolution in the sense that above each region of deficient density there will be a bulge or bump on the ellipsoid, and above each region of excessive density there will be a hollow [depression], relatively speaking. The bumps on this supposed earth will be the mountains, the plateaus, the continents; and the hollows [depressions] will be the oceans. The excess of material represented by that portion of the continent which is above sea level will be compensated for by a defect of density in the underlying material. The continents will be floated, so to speak, because they are composed of relatively light material; and, similarly, the floor of the ocean, on this supposed earth [will] be depressed because it is composed

density 2.83 represents more properly that of Reunion, and thus that of the floor of the Indian Ocean.

[39] Cf. Iddings, J. P.: "The Problem of Volcanism," p. 64, 1914.

[40] Dutton, C. E.: *Bull. Phil. Soc. Wash.*, xi, p. 51, 1889.

[41] For some references, see Iddings, *op. cit.*, p. 65.

[42] Hayford, J. F.: "The Figure of the Earth," p. 66, 1909; cf. Bowie, W.: *U. S. Coast and Geodetic Survey, Special Publication No.* 40, p. 7, 1917.

of unusually dense material. This particular condition of approximate equilibrium has been given the name isostasy."

As has been noted above, in the case of northern India, it has long been known, from pendulum determinations of gravity, that in many portions of the earth the observed force of gravity does not correspond with that calculated from the form of the geoid, after making corrections for the influence of topography (such as the attraction of near-by mountain masses), and for the elevation of the station above sea level. Thus, it has long been known that gravity is, on the whole, greater over the ocean than over land areas, and this has naturally been connected with the fact that the rocks of oceanic islands are mostly basaltic, and therefore heavy.

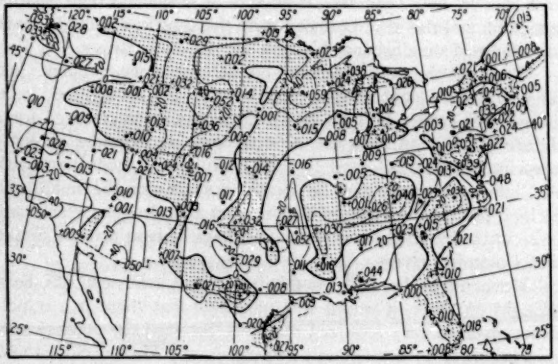
These departures from the normal are known as "anomalies." They may be either positive, when the gravity is above the normal, or negative, when it is below. The anomalies have been studied with great care and in great detail, especially by Hayford and Bowie in the area of the United States, who have invoked the theory of isostasy to account for them. This explanation is satisfactory to a very large extent, but, as Iddings says, "There remain anomalies of density which need to be accounted for." On the whole, however, it appears that the theory of isostasy "obtains for the major features of the earth's surface." It may be suggested here (though the matter cannot be discussed), that the discrepancies may be due, in part at least, to the fact that the geodesists have taken account of the rock densities only of those portions very near the surface, mostly soils and sedimentary rocks, and have neglected the deeper-lying portions. The average density assumed by geodesists for the surface rocks is 2.67, while, as we have seen, that of the igneous rocks of the earth's crusts is 2.77 or 2.75.

It will be seen that the idea of the continental masses being composed of light material, while the ocean floors are of heavy, is by no means new. Up to the present, however, there has been no quantitative verification of this, except for the few figures covering limited areas given by Iddings. The data given above, with the graphs, therefore, are of special interest as furnishing a first approximation to a knowledge of the actual specific gravities of the various portions of the earth's crust. It is evident that they are, on the whole, and even in considerable detail, quite in harmony with the theory of isostasy. Indeed, based as these

figures are on a large number of data (the largest by far yet available) from all parts of the earth, and showing such a complete harmony between average specific gravity and average elevation everywhere, they may fairly be said to be more than coincidental, and to constitute almost a conclusive proof of the general validity of the theory of isostasy.

One further point of agreement may be mentioned. In Fig. 6 is given a map of the United States reproduced by Barrell [43] from Bowie, showing the distribution of the gravity anomalies

FIG. 6.




Dotted portions: Areas of positive anomaly, excess of mass. Due to surface too high or density of upper crust above mean.
Blank portions: Areas of negative anomaly, deficiency of mass. Due to surface too low or density of upper crust below mean. Gravity anomalies in the United States.

over the United States. Let us compare this with the description of the comagmatic regions of the United States (page 796). In Fig. 5 the dotted areas are those of positive anomaly (excess gravity), while those not dotted are of negative anomaly (deficient gravity).

In the extreme northeast is a small area of positive anomaly about the Adirondacks, which corresponds with the small comagmatic outlier there of the Canadian Shield, of which the rocks are above the average in specific gravity. The greater part of Maine, with its granites, of low specific gravity, shows negative anomaly, and this area is continued down along the Appalachian

[43] Barrell, J.: *Jour. Geol.*, xxii, p. 153, 1914.

region, the rocks of which are of general low specific gravity. The small areas of positive anomaly in eastern Massachusetts, Connecticut, and New Jersey, may probably be connected with the extensive Triassic flows of heavy diabase and basalt which are so common here. Around the Lake Superior region we find an area of very high positive anomaly, which corresponds with the high specific gravity of the rocks of this region. On the other hand, in the Missouri-Arkansas-Oklahoma region is an area of pronounced negative anomaly, which corresponds with the low specific gravity of the region of the Ozark uplift. East of this, in Kentucky and Tennessee, is an area of somewhat high positive anomaly, and this is in harmony with the supposition made above that the rocks underlying this area are decidedly heavy.

Towards the west the relations become more complex, just as is the geological structure and as are the comagmatic relations. We cannot here go into details, which are reserved for a future publication, but the general area of negative anomaly covering the Colorado and Nevada-Utah plateau, consonant with the light rocks here, may be pointed out, as also the small area of high positive anomaly in Washington and Oregon, which may be connected with the very extensive flows of basalt of the Snake and Columbia rivers.

More might be said on this topic, but sufficient has been brought out here, it would seem, to show that there is a coincidence, even to very localized relations, between the average specific gravities of comagmatic regions in the United States and their gravity anomalies. Also, they appear to be too concordant to be the result of chance, so that we are justified in assuming that the two are causally related and that the theory of isostasy is thus justified.

## SUMMARY.

After brief consideration of the interior of the earth, the general characters of igneous rocks are discussed, and the presence of water vapor and other gases in the magma, and its analogy with a salt solution, are pointed out. In the discussion of the mineral characters of rocks, stress is laid on the fact that the number of essential rock-forming minerals is very small. These are mostly silicates of Al, Fe, Mg, Ca, Na, and K. Any two or more of these minerals (with two exceptions) may occur together and in any proportions.

The chemical characters of igneous rocks are summarized, and the ranges and maxima of the various constituents are given. The average igneous rock is considered and, after some discussion of the sources of error involved in the calculation, a new average in terms of oxides (based on 5179 analyses) is given. The average rock is shown to be approximately a granodiorite.

The average composition of the earth's crust in terms of elements is also given. Twelve elements (O, Si, Al, Fe, Ca, Na, K, Mg, Ti, H, P, and Mn) make up 99.61 per cent. of the crust.

The elements are referred to two main groups in the periodic table: (1) The petrogenic elements, characteristic of, and most abundant in, the igneous rocks, of low atomic weight, and occurring normally as oxides, silicates, chlorides, and fluorides; (2) the metallogenic elements, rare or absent in igneous rocks, but occurring as ores, of high atomic weight, and forming in nature "native" metals, sulphides, arsenides, bromides, etc., but not primarily oxides or silicates. The suggestion is made that beneath the silicate crust of petrogenic elements is a zone essentially of nickel-iron, and beneath this a central core of the metallogenic elements. This vertical distribution is in accord with Abbot's views as to the distribution of the elements in the sun.

In igneous rocks and minerals the elements show a correlation, in that certain of them are prone to occur with others, and a similar, limited correlation is apparently true of the animal and vegetable kingdoms.

The idea of "comagmatic regions," that is, the distribution of igneous rocks in regions of chemically related magmas, is discussed, and some of these are briefly described.

The calculation of rock densities from their chemical composition is discussed, and the average chemical compositions and densities of the continental masses and oceanic floors are given. It is shown by these that the average densities of the continents, ocean floors, and various smaller regions of the earth stand in inverse relation to their elevations. The bearing of this relation of average density and elevation on the theory of isostasy is pointed out, and it is shown that the data presented are confirmative of the theory.

GEOPHYSICAL LABORATORY,
CARNEGIE INSTITUTION OF WASHINGTON,
June 20, 1920.

**The Effect of Temperature upon the Transmission of Infra-red Radiation Through Various Glasses.** GEORGE ROSENGARTEN. (*Phys. Rev.*, Sept., 1920.)—The radiation from a tungsten lamp was spread into a spectrum by the use of two rock salt prisms. When a thermopile joined to a galvanometer receives a part of such spectrum a deflection of this instrument follows. The introduction of a plate of glass into the path of the ray of light was, of course, followed in all cases by a notable reduction in the deflection of the galvanometer. The piece of glass was placed inside of an electric heater so that its temperature could be varied at will and the deflection corresponding to each temperature was noted. Blue-purple, blue-green and other colored glasses were examined for wave lengths from .001 to .005 mm. with a variation of temperature from 40° C. to 46° C. The conclusion reached is that "to within 8 ± per cent. there is no change in the transmission of the infra-red radiation through the glasses examined."

G. F. S.

**The Separation of the Isotopes of Mercury.** J. N. BROENSTED and G. HEVSEY. (*Nature*, Sept. 30, 1920.)—Writing from the Physico-Chemical Laboratory of the Polytechnic High School, Copenhagen, the authors announce that they have succeeded in separating partially one of the isotopes of mercury from the other. In one set of experiments 40 g. of the liquid was taken and evaporated at low pressure until one-seventh of it had passed off. The mercury vapor was condensed and its specific gravity was found to be .999980 of that of ordinary mercury, while the ratio for the part of the liquid which did not evaporate was 1.000031. This difference results from the rate of evaporation being greater for the isotope of smaller atomic weight. "After the separation every portion was distilled again in the ordinary way and the density measured after each distillation. No difference was found between these measurements, the error of measurement of density being less than one in a million." G. F. S.

**Peat and Its Uses.** (*U. S. Geological Survey Press Bulletin No. 459*, November, 1920.)—Devotees of the automobile and motor-boat will be glad to know that successful experiments have been made in Sweden in extracting wood alcohol from peat. The process as reported by the commercial attaché at Copenhagen, Denmark, is described in a report on peat in 1919 recently issued by the Survey. Interesting instances of the use of peat as a fuel are given in this report. A coastwise steamship company of Norway, for example, during the coal shortage, was enabled by the use of peat fuel to keep up its full sailing schedule.

Peat is used also in making up fertilizers and in preparing concentrated food for stock. Last year 69,197 tons of peat, valued at $705,532, was produced in the United States. This was a decided decrease from the production of 1918, although the peat deposits in this country are extensive.

# NATURE AND EXPLANATION OF THE "MOTOR EFFECT" IN THE AJAX-WYATT FURNACE.*

BY

**E. F. NORTHRUP, Ph.D.,**

Palmer Physical Laboratory, Princeton University. Member of the Institute.

DR. CARL HERING read a paper May 3, 1907, before the American Electrochemical Society, on the limitations of resistance furnaces due to the "Pinch" phenomenon, which he was the first to observe in electric furnaces. He read another paper May 6, 1909, before the same society, which he entitled, "The Working Limit in Electric Furnaces, Due to the 'Pinch' Phenomenon." The writer of the present article presented a paper at the New York meeting of the Physical Society, March 2, 1907, which was published in the *Physical Review*, in June, 1907. The paper was entitled, "Some Newly Observed Manifestations of Forces in the Interior of an Electric Conductor."

Up to the time these papers were published few if any recognized that the passage of a high-density current through a liquid conductor produced motions in the liquid. It has since come to be recognized that when a high-density current, direct or alternating, flows in a liquid conductor, motions of the liquid of considerable magnitude are *always* produced.

The following analysis considers the nature and cause of the fluid motion which occurs at the apex of a fluid triangular conductor. The Ajax-Wyatt furnace makes use of the fluid motion thus produced to circulate molten brass in the triangular-shaped resistor employed. This furnace is now so well known and has been so fully described elsewhere, that further reference to it will not here be made.[1]

---

* Communicated by the Author.

[1] For a description of this furnace see "The Melting of Brass in the Induction Furnace," by G. H. Clamer, *Jour. of the American Institute of Metals*, vol. xi, No. 3.

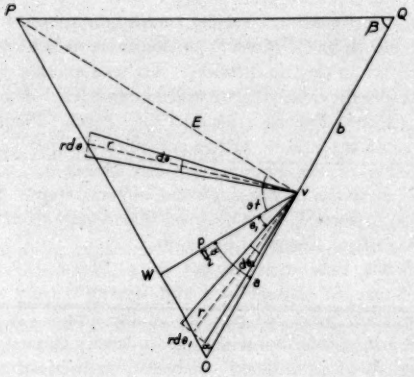
## SECTION I.

**DERIVATION OF FORMULA FOR CALCULATING THE MECHANICAL INTERACTION BETWEEN ELECTRIC CONDUCTORS WHICH FORM A TRIANGULAR CLOSED CIRCUIT.**

Let a current $i$ be induced in the circuit (see Fig. 1).

Let $H_t$ = the strength of the magnetic field at a point $v$ on the conductor $O-Q$ which results from the conductors $O-P$ and $Q-P$.

FIG. 1.




Let $r$ = the distance from $v$ to any point on the conductor $O-P$.

Let $d\theta$ = an elementary angle having its apex at $v$ and its base on $O-P$.

Then:

$$dH_t = \frac{ird\theta}{r^2} = \frac{id\theta}{r} \qquad (1)$$

as

$$p = r\cos\theta = a\sin\alpha, \; r = \frac{a\sin\alpha}{\cos\theta},$$

hence

$$dH = \frac{i}{a \sin \alpha} = \cos \theta \, d\theta \tag{2}$$

$$H_t = \frac{i}{a \sin \alpha} \int_0^{\theta_t} \cos \theta \, d\theta = \frac{i}{a \sin \alpha} \sin \theta_t \tag{3}$$

$H_1$ is the magnetic force at $v$ contributed by the length $W - P$ of the side $O - P$.

Let $H_2$ be the magnetic force at $v$ contributed by the length $O - W$ of the side $O - P$.

Then, as above:

$$dH_2 = \frac{i}{a \sin \alpha} \cos \theta_1 \, d\theta_1$$

where $\theta_1$ is the angle $r$ makes with $p$:

$$H_2 = \frac{i}{a \sin \alpha} \int_0^{\frac{\pi}{2} - \alpha} \cos \theta_1 \, d\theta_1 = \frac{i}{a \sin \alpha} \sin \left( \frac{\pi}{2} - \alpha \right) = \frac{i}{a} \frac{\cos \alpha}{\sin \alpha}$$

or

$$H_2 = \frac{i}{a \tan \alpha} = \frac{i}{a} \operatorname{cotan} \alpha \tag{4}$$

The total magnetic force at the point $v$ contributed by the side $O - P$ is $H = H_1 + H_2$, or

$$H = H_2 + H_1 = \frac{i}{a \sin \alpha} (\cos \alpha + \sin \theta_t) \tag{5}$$

To find the force $H^1$ contributed by the side $Q - P$, we merely have to change in Eq. (5) $a$ into $b$ and $\alpha$ into $\beta$, when we have:

$$H_t = H + H^1 = i \left[ \frac{1}{a \sin \alpha} (\cos \alpha + \sin \theta_t) + \frac{1}{b \sin \beta} (\cos \beta + \sin \theta_t') \right] \tag{6}$$

To be able to obtain numerical results, it is necessary to express $\sin \theta_t$ and $\sin \theta_t'$ in terms of the known quantities $a$, $L$, $\alpha$ and $b$, $S$, $\beta$.

This may be done as follows:

Call $E$ the distance from $v$ to $P$.

$p = E \cos \theta_t = a \sin \alpha$, hence

$$\cos\theta_t = \frac{a\sin\alpha}{E} \tag{7}$$

$$\cos^2\theta_t = \frac{a^2\sin^2\alpha}{E^2},$$

and

$$1 - \cos^2\theta_t = \frac{E^2 - a^2\sin^2\alpha}{E^2},$$

or

$$\sin^2\theta_t = \frac{E^2 - a^2\sin^2\alpha}{E^2} \tag{8}$$

Also,

$$E^2 = (L - a\cos\alpha)^2 + p^2 = (L - a\cos\alpha)^2 + a^2\sin^2\alpha$$

from which we find:

$$E^2 = L^2 - 2aL\cos\alpha + a^2 \tag{9}$$

Putting this value of $E^2$ in (8), we derive finally:

$$\sin\theta_t = \frac{L - a\cos\alpha}{[a^2 + L(L - 2a\cos\alpha)]^{1/2}} \tag{10}$$

By changing $L$ into $S$, $a$ into $b$, and $\alpha$ into $\beta$ we also have:

$$\sin\theta_t' = \frac{S - b\cos\beta}{[b^2 + S(S - 2b\cos\beta)]^{1/2}} \tag{11}$$

Place these values of $\sin\theta_t$ and $\sin\theta_t'$ in Eq. (6) and we obtain:

$$H_t = i\left\{\frac{1}{a\sin\alpha}\left[\cos\alpha + \frac{L - a\cos\alpha}{[a^2 + L(L - 2a\cos\alpha)]^{1/2}}\right] + \frac{1}{b\sin\beta}\left[\cos\beta + \frac{S - b\cos\beta}{[b^2 + S(S - 2b\cos\beta)]^{1/2}}\right]\right. \tag{12}$$

Equation (12) expresses the magnetic force at any point $v$ on the side $O - Q$ of triangular circuit in which this force is due to the current in the sides $O - P$ and $Q - P$.

We shall treat the case in which the triangle is equilateral. Then $\beta = \alpha$ and $S = L$, and Eq. (12) becomes

$$H_t' = \frac{i}{\sin\alpha}\left[\frac{1}{a}\left(\cos\alpha + \frac{L - a\cos\alpha}{\sqrt{a^2 + L(L - 2a\cos\alpha)}}\right) + \frac{1}{b}\left(\cos\alpha + \frac{L - b\cos\alpha}{\sqrt{b^2 + L(L - 2b\cos\alpha}}\right)\right. \tag{13}$$

The *mechanical* force which acts to move an element of length $\Delta a$ of a conductor carrying a current $i$ acts at *right angles* to the length of the element $\Delta a$ and is numerically equal to the strength of the magnetic field, to the current $i$ carried by the element of the conductor, and to the length of the element. Or:

$$\Delta F = i \Delta a H, \tag{14}$$

(Consult Maxwell: "Treatise on Electricity and Magnetism." Vol. II, Arts. 522, 597, 507.)

Put the value $H_1'$ from Eq. (13) in Eq. (14) for $H_1$ and we obtain the mechanical force which acts to move at *right angles to itself* the element of the conductor, $\Delta a$. Thus:

$$F' = \frac{i^2 \triangle a}{\sin \alpha} \left[ \frac{\cos \alpha}{a} + \frac{L - a \cos \alpha}{a \sqrt{a^2 + L (L - 2a \cos \alpha)}} + \frac{\cos \alpha}{b} + \frac{L - b \cos \alpha}{b \sqrt{b^2 + L (L - 2b \cos \alpha)}} \right] \tag{15}$$

For convenience call:

$$\frac{i^2 \triangle a}{\sin \alpha} = C; \quad \frac{\cos \alpha}{a} = A;$$

$$\frac{L - a \cos \alpha}{a \sqrt{a^2 + L (L - 2a \cos \alpha)}} = B; \quad \frac{\cos \alpha}{b} = A';$$

$$\frac{L - b \cos \alpha}{b \sqrt{b^2 + L (L - 2b \cos \alpha)}} = B'$$

Then write:

$$F' = C (A + B + A' + B') \tag{16}$$

The two terms $A$, $B$, express the contribution to the force given by the side $O - P$; and the two terms, $A'$, $B'$, express the contribution to the force given by the side $Q - P$. There is no action to move the element $\Delta a$ by the side $O - Q$.

**NUMERICAL RESULTS CALCULATED BY THE AID OF EQUATIONS (15) AND (16).**

We assume the following values for the constants:

$i$ = 800 electromagnetic C.G.S. units = 8000 amperes.
$i^2$ = 64 x $10^4$; $\triangle a$ = unit of length = 1 cm.
$L$ = 40 cm.; $\alpha = \beta = 60° = \frac{\pi}{3}$ radians.

The force is calculated for values of $a$ and $b$ given in the table below:

| $a$ cm. | $b$ cm. | $A$ | $B$ | $A'$ | $B'$ | $C$ | $F'$ dynes | $F'$ kilograms |
|---|---|---|---|---|---|---|---|---|
| 2.5 | 37.5 | .200 | .4 | .0133 | .0146 | $739 \times 10^3$ | 463900 | 0.473 |
| 5.0 | 35.0 | .100 | .1988 | .0143 | .0170 | $739 \times 10^3$ | 243870 | 0.249 |
| 7.5 | 32.5 | .0666 | .1313 | .0153 | .0199 | $739 \times 10^3$ | 172200 | 0.176 |
| 10.0 | 30.0 | .050 | .0971 | .0167 | .0231 | $739 \times 10^3$ | 138100 | 0.141 |
| 12.5 | 27.5 | .040 | .0762 | .0181 | .0269 | $739 \times 10^3$ | 119000 | 0.121 |
| 15.0 | 25.0 | .0333 | .0619 | .0200 | .0314 | $739 \times 10^3$ | 108300 | 0.110 |
| 20.0 | 20.0 | .025 | .0433 | .0250 | .0433 | $739 \times 10^3$ | 101000 | 0.103 |

## SECTION II.

### PRESSURE PRODUCED IN A NARROW STRIP WHICH CARRIES AN ELECTRIC CURRENT.

Let $a-b$ (Fig. 3) be a plane on the middle line through the strip normal to its greater width and parallel with its length, assumed very great as compared with the other dimensions of the strip.

The force of attraction of a linear conductor of infinite length carrying a current $i_e$, upon a unit length of a parallel conductor carrying a current $i$ and distant $r$ from the first, is:

$$F = \frac{2ii_e}{r} \tag{1}$$

(Maxwell, "Treatise on Electricity and Magnetism." Vol. II. Art. 495.)

Let $I$ = total current carried by the strip.
$W$ = thickness of strip.
$R_t = 2R$ = width of strip.

The current $i_l$, carried by a portion of the strip of width $2r$ is:

$$\frac{i_l}{I} = \frac{2r}{2R}$$

Hence:

$$i_l = \frac{r}{R} I = \frac{2r}{R_t} I \tag{2}$$

The current carried by a strip of width $dr$ is given by:

$$\frac{di}{{}^1/_2 I} = \frac{dr}{R}$$

Hence:

$$di = \frac{I}{2R}\,dr = \frac{I}{R_t}\,dr \tag{3}$$

The force which attracts a unit length of the element $dr$ carrying the current $di$ is:

$$dF = -\frac{2di\,i_t}{r} = -\frac{2 \cdot I\,\frac{dr}{R_t} \cdot 2\,\frac{r}{R_t}\,I}{r} = -\frac{4I^2}{R_t^2}\,dr \tag{4}$$

The minus sign signifies that the force diminishes as $r$ increases and that it acts toward the plane $a - b$.

The intensity of the force, or force per unit area, is:

$$\frac{dF}{W} = dg = -\frac{4I^2}{WR_t^2}dr \tag{5}$$

The intensity of the force at distance $r$ from the plane $a - b$ is:

$$g = -\frac{4I^2}{WR_t^2}\int_r^{\frac{1}{2}R_t} dr = \frac{2I^2}{WR_t^2}\,(2r - R_t) \tag{6}$$

The pressure, which is directed chiefly toward the plane $a - b$ is, on the surface of this plane:

$$g_p = -\frac{2I^2}{WR_t} \tag{7}$$

$g_p$ is the pressure per unit area which acts upon the median plane $a - b$. In calculating this pressure, if $I$ is taken in C.G.S. electromagnetic units, and $W$ and $R_t$ in centimetres, $g_p$ will be in dynes per square centimetre.

### NUMERICAL CALCULATION OF THE PRESSURE.

For constants choose:

$I$ = 800 electromagnetic C.G.S. units = 8000 amperes.
$W$ = 1.5 c.m.; $R_t$ = 7.6 c.m.; $I^2 = 64 \times 10^4$

Then, by equation (7), the pressure at the plane $a - b$ is:

$$g_p = -\frac{2 \times 64 \times 10^4}{1.5 \times 7.6} = -112280 \text{ dynes per cm.}^2$$

the minus sign meaning that this force is directed toward the median plane.

The *total* force which acts per centimetre length of the strip on the median plane is:

$$112280 \times 1.5 = 168420 \text{ dynes.}$$

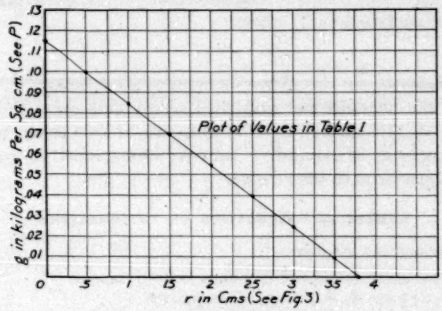
Expressed in kilograms these pressures are, respectively, 0.1145 and 0.1717.

By means of equation (6) we find the intensity of the force to have the following values at distances $r$, given in column 1, in the table below:

TABLE I.

| $r$ in cm. | $g$ in dynes | $g$ in kilos | $Wg = 1.5g$ in kilos | $g$ in lbs. per sq. inch |
|---|---|---|---|---|
| 0 | 112280 | 0.1145 | 0.1717 | 1.629 |
| 0.5 | 97530 | 0.0995 | 0.1492 | 1.416 |
| 1.0 | 82760 | 0.0840 | 0.1260 | 1.195 |
| 1.5 | 68000 | 0.0693 | 0.1040 | 0.986 |
| 2.0 | 53200 | 0.0543 | 0.0814 | 0.773 |
| 2.5 | 38420 | 0.0392 | 0.0588 | 0.558 |
| 3.0 | 23650 | 0.0241 | 0.0361 | 0.343 |
| 3.5 | 8868 | 0.0090 | 0.0135 | 0.128 |
| 3.8 | 0 | 0 | 0 | 0 |

CURVE 1.




## SECTION III.

### NECESSARY ASSUMPTIONS MADE IN DERIVING EQUATION (15), SECTION I.

In order to derive a formula to calculate the forces set up by electromagnetic action upon one small portion of a conductor by the remainder of the circuit, it is necessary to assume the circuit

to have some simple geometrical form. The geometrical form which most closely approximates the actual case which we have to consider and which can be handled mathematically, is an equilateral triangle.

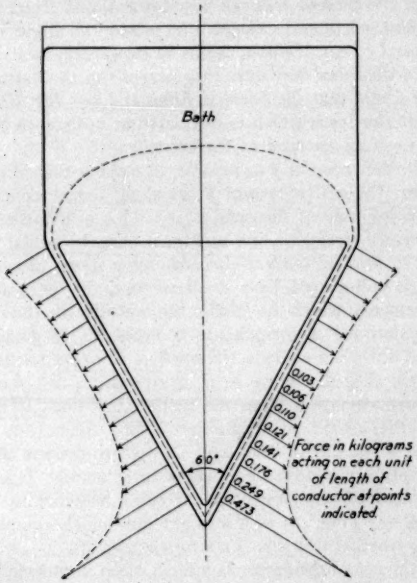
If in equation (6) we place $a = O$ and $b = 40$ the equation gives $H_t$ equals infinity. This does not show that the equation is unsuited for calculating the magnetic force, because in the actual case the current does not flow toward, and from, a mathematical point where two straight lines meet in an angle, but flows round a bend of considerable radius of curvature; and if it were possible to take this curvature into account in the calculation it would be found that the force is finite and not far different in value from the force which is calculated at a distance of 4 or 5 centimetres from an apex of the triangle.

It is further necessary to assume, to make a calculation practicable, that the entire current flows along a mathematical line located on the axis of the conductor. This is indicated by the triangle drawn in dotted line in Fig. 2. In the actual case the force which acts on current-elements lying along the inside of the triangle will exceed, by a small amount, the force acting on current-elements which lie along the outside of the triangle. While in deriving a formula, it is necessary to assume that the current flows in a triangle, the small value of the terms, $A'$, $B'$, as compared with the terms $A$, $B$, in equation (16), shows that only a small deviation from the actual conditions is made by assuming the circuit of triangular form. Thus, for $a = 2.5$, $A' + B' = 0.0279$, and $A + B = 0.6$. $A' + B'$ contribute thus only about 4.4 per cent. toward the whole force acting It is, therefore, of small consequence upon the result whether we assume the side $Q - P$ (Fig. 1) is a straight line which completes the triangle or whether this side is a semi-circular arc.

Since the actual conductor is 7.6 cm. deep in a direction perpendicular to the plane of the drawing (Fig. 2) and only 1.5 cm. wide, the *difference* between the force acting upon the outside triangular surface and the force acting upon the inside triangular surface is very small. At a distance of 7.5 cm. from the apex of the angle shown in dotted line, I estimate this difference to be about 0.036 kilogram, or 0.00473 kilogram per cm. The difference in pressure between the outside edge and the median plane of the conductor, in the direction vertical to the plane of the

drawing, due to the mutual attraction of the current elements in the conductor, is 0.1145 kilogram per cm. Thus the effective pressure-difference over a plane *normal* to the length of the conductor, due to the mutual attraction of current-elements, is in the neighborhood of 24 times the pressure-difference over the

FIG. 2.




plane which would be set up by the mutual repulsion between the two limbs of the conductor at a distance of 7.5 cm. from the apex of the triangle represented in dotted line in Fig. 2.

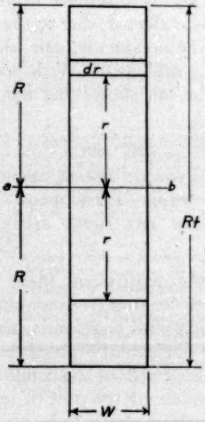
The forces of mutual repulsion between any two sides of the triangle are greatly modified in the actual case by the field which results from the leakage lines from the transformer. This circumstance alone will greatly modify any estimate based on calcu-

lation of the repellent forces and the pressure-difference forces produced by the action of one limb of the circuit upon another.

**NECESSARY ASSUMPTIONS MADE FOR DEDUCING EQUATION (6), SECTION II.**

In deducing Eq. (6) it has been assumed that the force of attraction which acts upon a strip $dr$ infinitely thin, carrying a current element $di$, acts only toward, and normal to, the median plane $a-b$ (Fig. 3). In reality, a force of attraction is also

FIG. 3.




exerted between opposite ends of such strip. Thus the actual force on the current element $di$ is the resultant of two components, one very much smaller than the other. If the integration were made for this resultant force (a very complicated process) it would be found that the pressure is not quite normal on the plane $a-b$ but is directed slightly toward the centre longitudinal axis of the conductor. If the conductor had a square cross-section, so that $W = Rt$, then there would be as much pressure at its centre directed along the horizontal as along the vertical (Fig. 3). The intensity of the force at the centre would be then more nearly represented by the formula, $g = \frac{I^2}{S}$, where $S$ is the area of the

cross-section of the conductor. However, if $W$ is small as compared with $R_t$, as in the actual case, Eqs. (6) and (7) give the intensity of the force to a very close approximation and may be used safely to calculate the pressure per unit area set up by the mutual attraction of the current elements in the single conductor.

As is shown, tendency to fluid-motion, which results from mutual attraction of current elements, depends upon difference in pressure between two points on a plane through the conductor normal to its length. This difference in pressure between centre and edge of the strip-conductor, due to the mutual attractions of current elements in the conductor, can be calculated with good precision by Eqs. (6) and (7). With 8000 amperes this pressure-difference will be, as shown, as much as 1.6 pounds per square inch.

## SECTION IV.

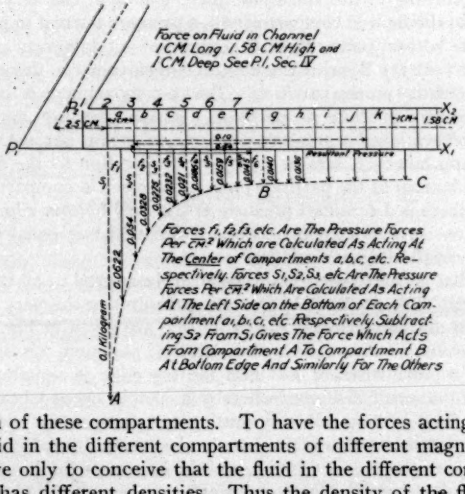
### NATURE OF THE HYDRODYNAMIC FORCES DEVELOPED BY THE ELECTRODYNAMIC FORCES WHICH ARISE FROM THE ACTION OF THE ENTIRE CIRCUIT UPON ANY SHORT SECTION OF THE LENGTH OF THE CONDUCTOR.

To understand the nature of these forces, conceive the V-shaped conductor to lie in a vertical plane, with one limb of the conductor in a horizontal position, as drawn in Fig. 4. Conceive further that the horizontal limb of the conductor is divided by planes normal to the length of the conductor-limb into sections separated from each other by the unit of length; in this case one centimetre. Conceive these planes to be made of thin, solid-metal sheet. The conductor limb will now be divided up into sections *a, b, c, d,* etc., which are 1 cm. wide, 1.58 cm. high and 7.6 cm. deep. Take the data from Fig. 2, Section I, and draw a curve *A, C, B* (Fig. 4), such that lines drawn from the middle of the lower side of each of the sections *a, b, c, d,* etc., *normal* to this lower side and terminating in the curve, represent by their length and direction the magnitude and direction of the average pressure exerted upon the fluid contained in a section 1 cm. long, 1.58 cm. high, and *1 cm.* deep. Thus, these lines represent both in respect to magnitude and direction the normal pressure *per cm.*$^2$ upon the lower side $P_1 - X_1$ of the limb of the conductor. In Fig. 4 these lines have been drawn so that a length of 1 cm. represents a

pressure of 0.01 of a kilogram. If we conceive the pressure acting upon the fluid in each compartment, *a*, *b*, *c*, *d*, etc., to be uniformly distributed over each horizontal layer of the fluid in each compartment, the fluid will be everywhere in equilibrium on account of the reaction of the walls of the compartment, and it will be quiescent.

We may, however, conceive that the forces represented as acting upon the fluid in compartments *a*, *b*, *c*, *d*, etc., are due *not* to an electric current, but *to the action of gravity* upon the fluid

FIG. 4.


Force on Fluid in Channel
1 CM. Long 1.58 CM. High and
1 CM. Deep See P1, Sec. IV
Resultant Pressure
Forces f1, f2, f3, etc. Are The Pressure Forces Per CM.² Which are Calculated As Acting At The Center of Compartments a, b, c, etc. Respectively. Forces S1, S2, S3, etc. Are The Pressure Forces Per CM.² Which Are Calculated As Acting At The Left Side on the Bottom of Each Compartment a1, b1, c1, etc. Respectively. Subtracting S2 From S1 Gives The Force Which Acts From Compartment A To Compartment B At Bottom Edge And Similarly For The Others
0.1 Kilogram

in each of these compartments. To have the forces acting upon the fluid in the different compartments of different magnitudes, we have only to conceive that the fluid in the different compartments has different densities. Thus the density of the fluid in compartment *a*, for example, to give a pressure of 0.1 kilogram on the lower side of this compartment, would be:

$$\delta_a = \frac{\text{Pressure per cm.}^2}{\text{Acceleration of gravity x volume}} = \frac{100\ (980)}{980 \times 1^2 \times 1.58} = \frac{100}{1.58}$$

The density for compartment *b* would then be:

$$\delta_b = \frac{54}{100}\,\delta a,$$

and similarly for the other compartments.

With these graded densities of the fluid in the different compartments, the pressure-forces will be in all respects identical to the forces which are produced on a uniform fluid by the repellent action of the current.

The well-known laws of the pressure of a fluid in a vessel are:

1. "The pressure in each layer is proportional to the depth."

2. "With different liquids and the same depth, the pressure is proportional to the density of the liquid."

3. "The pressure is the same at all points of the same horizontal layer." (*Ganot's Physics*, art. 98.)

According to this third law, there will be at the very *bottom* layer of the fluid in compartment *a*, a pressure normal to partition *3* at its bottom point, equal to a force of 0.1 kilogram per cm.$^2$ At the *very top* layer of the fluid in compartment *a*, there will be zero pressure on the partition. But in compartment *b* there will also be zero pressure on partition *3* at the *very top*, and at the very botom layer a pressure of 0.054 kilogram per cm.$^2$ This pressure, however, acts in the *opposite* direction to the pressure on the bottom of the partition, which has place in compartment *a*. Thus there is a *resultant* pressure at the *very bottom edge* of the partition between compartments *a* and *b* which is equal to $0.1 - .054 = 0.046$ kilogram per cm.$^2$

What is true of sections *a* and *b* can be applied to all the other sections. The resultant hydrostatic pressures at the very bottom edge of all the partitions *3*, *4*, *5*, *6*, etc., are given in Fig. 4.

Assume now that the bottom edge of partition *3* is suddenly raised a small distance and that the top edge is simultaneously lowered a small distance, whereby a slot is opened below and above the partition, suddenly putting compartments *a* and *b* into communication. It is plain that the denser fluid in *a* would flow through the bottom slot into *b*, and the less dense fluid in *b* would flow in the opposite direction through the upper slot into *a*. This fluid motion would continue if *gravity alone* were acting, until the fluid in *a* is evenly divided between compartments *a* and *b*, and likewise with the fluid in *b*.

In the case of the forces set up by electromagnetic action, the fluid is in reality all of the same density, but immediately any fluid leaves a compartment and moves to the right, it is subjected to a *less* force; that is, it acts as if, in the gravitational case, it lost density; and any fluid which enters a compartment

toward the left is subjected to a greater force; that is, it acts as if, in the gravitational case, it gained density. The result of this is that, in the actual case, where the normal pressure-forces upon the fluid are due to the repulsion of the current in the rest of the circuit, and where these pressure-forces diminish in magnitude as the distance along the conductor from the apex of an angle increases, the fluid is acted upon by hydrodynamic forces which not only cause it to circulate, but *to circulate continuously*.

We now only have to conceive that all the partitions are removed, and we shall have, in the actual case, a stream of the fluid moving along the side $P_1 - X_1$ of the conductor, and a returning stream moving along the side $X_2 - P_2$. The total hydrodynamic pressure-difference between any two points along the conductor which acts to maintain this flow is given by taking the difference of the normal pressures produced by electrodynamic forces between these two points. This pressure-difference, for example, between a point located on the middle of the bottom of compartment *a* and on the middle of the bottom of compartment *r* is:

$$0.1000 - 0.0126 = 0.0874 \text{ kilogram per cm.}$$

The power expended in moving the fluid from one point to another along the conductor will be $P = (F_1 - F_2)V$, namely, equal to the difference in the pressure between two points times the velocity, or grams per second, of fluid which crosses in one direction any section of the conductor between the two points where the pressures are $F_1$ and $F_2$. This power is, of course, derived from the transformer, and, due to the motion of the fluid, the conductor will develop a back E.M.F., so that a greater E.M.F. will be required to maintain the current than would be the case if the conductor were a solid instead of a fluid which can move, and in moving do work.

We can now give a definition of the "motor-effect," which is a reality.

When by any geometrical disposition whatever of an electric circuit, in which the conducting material is a fluid capable of free motion, normally acting electrodynamic forces arise in any section of the circuit which vary in magnitude from one point to another over a length measured along the axis of the conductor, there also arise hydrodynamic forces which can impress

motions on the fluid substantially parallel to the longitudinal axis of the conductor.

Actual fluid motions which result from these hydrodynamic forces are designated, for brevity, *" The Motor-effect."*

The magnitude of the forces which give rise to a motor-effect may be ascertained as given in Fig. 2. The motor-effect which results, namely, the velocity of the fluid considered both in magnitude and direction, produced by the forces, cannot be calculated because the resulting velocity, wholly in magnitude and partly in direction, depends upon influences which act to restrain the flow, such as frictional resistance, viscosity of fluid, density of fluid, etc.,—and these are unknown.

It is easy to see, however, that the forces which produce a motor-effect are greatest at, or very close to, the apex of an angle, and consequently we can assume safely that the motor-effect is here quite considerable. The forces to produce the motor-effect diminish as the bath is approached, at which place these forces are practically nil.

Since it has never been shown, experimentally, heretofore, that a motor-effect can be produced separate from and uninfluenced by any internal pressure-effect, apparatus was made by me and an experiment was performed with this apparatus which *conclusively* demonstrated that a motor-effect can be produced apart from any internal pressure-effect. The experiment was made, but is not further referred to here.

## SECTION V

### EFFECT OF THE MUTUAL ATTRACTIONS OF CURRENT-ELEMENTS IN PRODUCING FLUID-MOTION.

Because of the mutual attractions of the current-elements in the conductor, the electrodynamic forces act toward the axis in the case of a conductor of circular cross-section, and, for the most part, normal to a median plane parallel to the length of the conductor, in the case of a conductor having the form of a narrow strip. These electrodynamic forces, pressing the fluid toward an axis or a plane, give rise, in virtue of Pascal's Law (that " Pressure exerted anywhere upon a mass of liquid is transmitted undiminished in all directions, and acts with the same force on all equal surfaces and in a direction at right angles to those surfaces,"

—*Ganot's Physics,* art. 97), to hydrodynamical forces which act to urge the fluid, near the axis or median plane of the conductor, in a direction parallel to the length of the conductor. If the conductor increases in cross-section suddenly, as it does in our case where it enters the bath, the longitudinal hydrodynamic forces are unbalanced by any opposing force, and consequently the fluid near the centre of the conductor is urged in the direction of the expanded portion of the conductor. As the fluid, however, is held in a rigid casing, there is developed a return current of fluid which flows near and parallel to the walls of the casing. We shall call the hydrodynamical forces which arise from the action of the mutual-attraction of current-elements in the single conductor, the *Internal Pressure Forces,* and we shall call any motion of the fluid which results from the action of these forces, the *Internal Pressure-effect.* This latter has long been known as the "Pinch Effect" (which, if carefully considered, is a misnomer).

The internal pressure forces act throughout the length of a conductor and are directly proportional to the mean square of the total current carried by the conductor, whether this be alternating or direct, and are inversely proportional to the cross-section of the conductor. As a consequence of the fact that in the "Wyatt furnace" the conductor opens out suddenly into a metal pool or bath, the internal pressure-effect is very marked at this place, because here the internal pressure forces are unbalanced by oppositely-directed forces. *The motor-effect, on the contrary, is far more marked near the apex of the V-shaped conductor.*

As a consequence, we reach the important conclusion: that *the Wyatt furnace has the peculiar and very practically desirable feature of developing hydrodynamical forces of considerable magnitude,* which give rise to fluid motions particularly at three points: the apex of the V-shaped conductor, and the two points where this V-shaped conductor enters the pool or bath of molten metal. Furthermore, the upright position occupied by the furnace effectually prevents any rupture of the circuit by the internal pressure-forces, and, in addition, offers the best disposition for any circulation of fluid which might arise from the tendency of the more highly-heated and less-dense fluid to rise.

It should be noted in conclusion that both the motor-effect and the internal pressure-effect increase with the square of the

current, and thus any increase in current will cause these two effects to increase in the same ratio. In the Wyatt furnace, I estimate the internal pressure-force approximately 1.3 times the maximum hydrodynamical force which gives rise to the motor-effect. It is reasonable to suppose that the two effects are in approximately the same ratio as the two forces which produce these effects.

## SUMMARY AND CONCLUSIONS.

*Section I.*—A formula (page 821) is developed for calculating the mechanical forces which arise by the interaction of a triangular circuit carrying a current upon any short section of the circuit. Numerical results are tabulated (page 822) and values of the forces are given in a chart.

*Section II.*—Formulas are developed (page 823) for calculating the internal pressure-forces in a thin strip conductor. Numerical results are tabulated (page 824). A curve giving the pressure as a function of the distance from the axis of the strip is appended.

*Section III.*—The assumptions made in deriving Eq. (15), Section I, are considered; also the assumptions made for deducing Eq. (6), Section II.

*Section IV.*—The exact meaning and a definition of the *motor-effect* is given. It is shown also that such an effect actually exists and assumes considerable importance in the "Ajax-Wyatt furnace." A chart (Fig. 4) is given which shows the magnitude of the forces which produce the motor-effect. A new experiment is referred to which conclusively proved that the motor-effect not only exists, but may be manifested apart from, and independent of, any internal pressure-effect. The effect of the mutual attractions of current-elements in producing fluid motion are generally considered, and the *internal pressure-effect* is defined.

The important conclusion is reached that the Ajax-Wyatt furnace secures both the *internal pressure-effect* and the *motor-effect* in an advantageous way for obtaining practical results. It is further pointed out that the securing of these two effects at widely-separated sections of the conductor is a valuable and, as far as the writer knows, a novel result in furnace construction of the induction type.

# THE ANNEALING OF GLASS.* †

BY

L. H. ADAMS and E. D. WILLIAMSON.

Geophysical Laboratory, Carnegie Institution of Washington.

## THE PROPER PROCEDURE FOR ANNEALING GLASS.

In considering the annealing of glass from a practical point of view, we must take account of three factors: (1) The kind of glass; (2) the initial amount of strain; and (3) the lower limit of strain, that is, the permissible amount of strain in the annealed glass. The rate of annealing, of course, depends on the chemical composition of the glass and also on the amount of strain. Furthermore, since the strain never actually disappears, but approaches zero asymptotically, the "time of disappearance" of strain at a given temperature will depend on the accuracy with which the strain can be measured; in other words, the annealing-time for a given temperature, or conversely, the annealing-temperature for a given time, will vary with the sensitivity of the strain-detector or with the amount of strain to be tolerated in the finished product.

*Standard of Annealing.*—The allowable strain will depend on the purpose for which the glass is to be used. Naturally, commercial glassware would require less complete removal of strain than lenses and prisms for optical instruments, but unfortunately little or no information is available for determining exactly the amount of strain which would be detrimental in the various kinds of glass articles. Twyman [44] proposes to call glass annealed when its internal stress has been reduced to 1/20 of the breaking stress and arbitrarily defines the annealing temperature as the temperature at which the given glass would lose in 3 minutes 95 per cent. of its original stress. The time required for the release of a given fraction of the stress is not, as Twyman apparently believed, independent of the initial stress, but nevertheless, Twyman's definition of "annealed" glass and of "annealing

* Communicated by Dr. Arthur L. Day, Director of Laboratory.

† Concluded from page 631, Vol. 190, November, 1920.

[44] *Loc. cit.*, p. 70.

temperature" may furnish the basis for a convenient standard for manufactured articles of ordinary glass.

For optical glass F. E. Wright proposed as an arbitrary standard that the maximum path difference in an annealed block of glass shall not exceed 20$\mu\mu$ per cm., basing this standard on the fact that the path difference per cm. in various samples of glass from Schott, Mantois, and Bausch and Lomb, examined by him, varied between 5 and 50$\mu\mu$. During the war the standard of 20$\mu\mu$ per cm. was adopted in the inspection of optical glass, and, in the absence of any definite knowledge of the amount of stress which causes warping of lenses and prisms with consequent impairment of the optical definition, some such arbitrary standard is necessary. Since it is a relatively simple matter to anneal glass so that the strain is much less than 20$\mu\mu$ per cm., it is believed by the authors that the standard of good annealing might well be placed at a still lower value, say 10$\mu\mu$ per cm. maximum path difference.[45] In a slab of glass viewed lengthwise the maximum path difference is at the two surfaces, and is usually about double the path difference along the middle. For slabs of glass (the form in which optical glass is usually supplied) the proposed standard of good annealing would limit the path difference along the *middle* to 5$\mu\mu$ per cm. Since the release of stress proceeds according to equation (7), it follows that the annealing time is influenced more by the lower than by the upper limit of its stress. Thus, the time required to reduce the path difference from 100 to 5 is only 8 per cent. longer than the time to reduce the stress from 50 to 5, while on the other hand, the necessary time for changing the stress from 50 to 10 is less than 45 per cent. as long. Since unannealed optical glass is seldom strained to an extent corresponding[46] to more than 50$\mu\mu$ per cm. (in the middle), it

[45] In a private communication to the authors, Dr. Wright states that the standard of 20$\mu\mu$ per cm. was placed at that value partly because under the stress of war conditions a fairly high limit was necessary. He also states that although glass with 20$\mu\mu$ of strain will make satisfactory optical instruments, his own experience has demonstrated that it is quite feasible to anneal glass so that the strain is much less than 20$\mu\mu$, and that, in agreement with us, he would place the standard of annealing for optical glass below the original value.

[46] A slab 2 cm. thick of any ordinary glass will show a path difference of 50$\mu\mu$ per cm. along the middle when cooled at the rate of about 230° C. per hour. In a preliminary note (*J. Opt. Soc. Amer.*, **4**, 213–223, 1920), the authors defined annealing time on the basis of limits which were 50$\mu\mu$ and 5$\mu\mu$.

is convenient, when dealing with *optical* glass, to define the *annealing time* at a given temperature as the time required for the path difference (as determined by observation at the centres of the polished ends) to change from 50$\mu\mu$ to the chosen lower limit, which it might seem should be 5$\mu\mu$, our arbitrary standard. In what follows, however, it will be shown that the best method for the annealing of optical glass in actual practice requires the stress to be reduced (at the annealing temperature) to 2.5$\mu\mu$ per cm., if the final amount in the cold glass is to be 5$\mu\mu$. In connection with the annealing of *optical* glass, therefore, we define the *annealing time* at a given temperature as the time required to reduce the stress from 50 to 2.5$\mu\mu$. Thus, for ordinary crown the annealing time at 573° is 2 minutes, and for light flint the annealing time at 395° is 5 hours (see Table V). The *annealing temperature* for a given annealing time is similarly defined.

*Annealing Range.*—One other commonly used term needs to be mentioned in this connection. The range of temperature in which annealing ordinarily occurs is sometimes called the annealing-range of the glass in question. This is necessarily a rather loose term, since glass anneals at any temperature. Just as a substance such as mercury has no real "volatilizing range" but has a finite vapor pressure at all temperatures (except, of course, above the very high critical temperature, liquid-vapor), so glass anneals at a finite rate at all temperatures; and just as the lower limit of the "volatilizing range" would depend on the precision of the apparatus for measuring vapor-pressure, so the lower limit of the annealing range would depend on the delicacy of the method used for detecting strain. Nevertheless, there is a certain range of temperature in which most of the important operations connected with glass-annealing are conducted and in which the release of stress proceeds at an easily measured rate. This range extends over about 150° and is limited at the upper end by the temperature at which strain as ordinarily measured is seen to disappear when the heating rate is the usual amount of a few degrees per minute. Solely for convenience in referring to temperatures in this region, we shall therefore arbitrarily define the *annealing range* of a particular glass as that 150° interval of temperature lying immediately below the temperature at which the annealing time is 2 minutes. For example, as may be seen from Table V, the "annealing range" of ordinary crown glass is 423 to 573°.

*The Necessary Conditions for Annealing.*—In removing the strain from unannealed glass two problems are presented: First, the removal of the internal stress, and second, the prevention of its return. The removal of the stress is a simple process merely involving the heating of the glass to a sufficiently high temperature; to prevent the return of stress on cooling, however, is a much more complicated matter. As has already been explained, the permanent stress acquired by a piece of glass when cooled is equal and opposite in sign to the temporary stress lost by viscous yielding when the glass is in the "annealing range" of temperature. In other words, if a certain part of a glass block shows a birefringence corresponding to a *thrust* of 10 kg/cm.$^2$, then while the glass was hot the *tension* in that part of the block must have been reduced by 10 kg/cm.$^2$ It is evident, therefore, paradoxical as it may seem, that in order to obtain the glass as free as possible from internal stress, it is necessary, while the glass is being cooled, to prevent the release of stress. The most obvious way in which to accomplish this is to reduce the amount of temporary stress, for, other things being equal, the less the stress the less will be lost. Hence the glass, after being maintained for a short time at the upper end of the annealing range, is cooled slowly so that the temporary stress introduced is comparatively small in amount; in fact, so small that even if it is all lost in cooling the final permanent strain will be within the allowable amount. This leads us to the:

*Older Method for Annealing.*—This method consists in heating the glass at a moderate rate to a temperature near the upper end of the annealing range. When the strain is judged to have disappeared, the heating is stopped, and the glass is then cooled slowly down to room temperature. For this method, the "annealing temperatures," that is, the maximum temperatures reached, need not be greater than those given in the fourth column of Table V. The permanent strain acquired by the glass will depend on the rate of cooling which for this method will necessarily be rather slow.

*Improved Method for Annealing.*—If instead of heating the glass to a relatively high temperature, we may hold it for a sufficient time at a temperature at which it takes several hours to anneal, we may then cool it fairly rapidly without causing the reappearance of permanent strain. The reason for this is as fol-

lows: The glass starts to cool, thus acquiring temporary strain, at a temperature at which the viscosity is high. Hence, although the temporary strain may be large, the amount lost during cooling will be comparatively small, and according to the theorem enunciated above, the permanent strain, being equal to the temporary strain lost in cooling, will also be small.

Thus, in contrast with the ordinary procedure of annealing glass for a short time at a high temperature, we may anneal it at a much lower temperature for a longer time; and, whereas, by the first method the subsequent cooling must be slow, by the second method it may be comparatively rapid.

This second method was used by the authors in annealing optical glass (over a hundred thousand pounds) at the Charleroi plant of the Pittsburgh Plate Glass Company, and the satisfactory results obtained indicated that this procedure possesses many advantages over the ordinary method. In the first place, it is much easier to hold an annealing kiln at a constant temperature than to cool it at a given constant rate. The newer method requires a fairly constant temperature for some hours, but the subsequent cooling takes place in a short time and does not need careful control, so that on the whole the chances of failure, particularly when unskilled workmen are employed to regulate the burners, are less than with the older method which involves the difficult operation of cooling at a constant slow rate. Another advantage of annealing at the lower temperature is that it minimizes the danger of overheating the glass. This is of especial importance when annealing valuable optical glass, large batches of which in several cases known to the authors have been spoiled, due to the fact that the temperature in the annealing furnace was accidentally allowed to become about 50° too high. Moreover, annealing at a low temperature not only saves time in heating up the glass to the annealing point, but also shortens somewhat the time of the actual annealing plus cooling, especially, as will be shown below, for larger blocks of glass. Finally, the danger of devitrification is obviously much less when the glass is annealed at a low temperature.

In this connection it may be mentioned that the procedure, sometimes followed, of heating glass to the upper end of the annealing range and then allowing it to "soak" at this temperature for several hours or over night, has no relation to the method

here advocated. When the glass has once been heated to the temperature at which it anneals almost instantly, nothing whatever is gained by holding it at the high temperature, and on the other hand, many of the disadvantageous features of the high-temperature method are correspondingly aggravated.

*Rate of Cooling and Its Dependence on the Method of Annealing.*—It follows from what has been said that there is a certain latitude allowable in the temperature at which glass may be annealed, and that the rate of cooling will be dependent on this temperature. The glass may be heated to its softening point and cooled at a rate which will cause the stress to be no more than the allowable amount, say, 5$\mu\mu$, or, on the other hand, the temperature may be raised only to the point where the original stresses require some hours to be released, after which the temperature is allowed to fall rapidly. It is of interest to compare the relative time consumed by these two methods in order to determine whether or not by the second method the rate of cooling can be great enough to compensate for the much longer time spent at the annealing temperature. The problem of calculating the proper rate of cooling is a complicated one, and rather than attempt its complete solution we shall consider several special cases. Arbitrarily dividing the problem into two parts: (I) the case of glass heated to a temperature near (or above) the upper end of the annealing range, and (II) the case of glass annealed at a much lower temperature, we shall calculate for a slab of glass 2 cm. thick the strain introduced by cooling in certain specified ways.

(I) In this case the glass is heated to a temperature at which the annealing constant $A$ is about 0.19. The annealing temperatures for the various glasses may therefore be taken from the fourth column of Table V. Temperatures higher than these are quite unnecessary, since with any ordinary heating rate the strain will have (sensibly) disappeared by the time this temperature has been reached. Supposing now that the strain has been removed from the glass, we shall proceed to calculate the strain introduced ($a$) by cooling at a constant rate, and ($b$) by cooling at a rate which increases in geometrical ratio.

($a$) When glass is heated to a relatively high temperature and then cooled at a constant rate, the final stress is exactly the same as the *temporary* stress which would be caused by *heating* at the same rate. The stress in the middle for an average glass is there-

fore (see equation 54a below) $F = 4.6\ ha^2$, and the birefringence is

$$\Delta n = 4.6\ Bha^2 = ch \qquad (10)$$

in which $a$ is the semi-thickness, $h$ is the cooling rate and $c$ is the equivalent of $4.6\ Ba^2$, and is thus a constant depending on the properties of the glass and the dimensions of the block. The values of $B$ for the more common optical glasses do not differ much from $2.8 \times 10^{-7}$. Hence for a slab 2 cm. thick $c$ is about 13 if $\Delta n$ be measured in $\mu\mu$ per cm. and $h$ be in deg. C. per minute. We are now able to calculate the allowable cooling rate for glass annealed by this method. Thus if the standard of annealing be taken as $5\mu\mu$ per cm. in the middle ($10\mu\mu$ at the surface), the allowable cooling rate from equation (10) would be $\frac{5}{13} = 0.385$ deg. per min. or 23 deg. per hr. Thus slabs of ordinary thickness when annealed by this method should be cooled at the rate of 23 deg. C. per hr. This rate varies somewhat with different kinds of glass, but except for the very heavy flints the divergence is not great. Furthermore, the rate of cooling is inversely proportional to the square of the thickness. Thus a lens blank 20 cm. thick would be cooled at a rate of $\frac{4}{400} \times 23 = 0.23$ deg. per hr., or 5.5 deg. per day, if not more than $5\mu\mu$ (in the middle) could be tolerated. With a limit of $25\mu\mu$ in the middle—50 at the surface—the corresponding rate by this method would be 27.5 deg. per day.

As a check on this calculation of the strain introduced by cooling at a given constant rate, the following experiment was performed: A slab of ordinary crown glass 2.2 cm. thick and 7.6 cm. long was heated in an electric furnace to a temperature considerably above the annealing range and cooled at a rate which, within the annealing range, was approximately constant and equal to about 4.3 deg. per min. When cold the glass was examined and the optical path difference along the middle found to be $415\mu\mu$, or $415/7.6 = 55\mu\mu$ per cm. According to equation (10) the birefringence should have been $4.6\ Bha^2$, or (since $B$ for ordinary crown glass is 2.57), $4.6 \times 2.57 \times 4.3 \times (2.2/2)^2 = 61\mu\mu$ per cm. The agreement is satisfactory when the difficulty of maintaining an accurately constant cooling-rate is taken into consideration.

It has generally been recognized that in cooling glass it is not necessary to continue the constant cooling-rate right down to

room-temperature and that the rate can therefore be increased somewhat in the later stages of cooling. As will be shown later, comparatively little strain can be introduced below what we have called the annealing range, and hence the cooling-rate below the annealing range is limited mainly by what the glass will stand without breaking and by the rapidity with which it is practicable to cool the furnace. The tensile strength of glass is usually about 600 kg/cm.$^2$, which corresponds (cf. equation 54b) to a cooling rate equal to $\frac{65}{a^2}$ deg. per min. For safety a much lower maximum rate would be chosen—say, $\frac{10}{a^2}$ deg. per min. Finally, then, to anneal by this method a 2 cm. slab ($a = 1$) of borosilicate glass, it would be heated to 599°, cooled at a rate of 23 deg. per hr. for 150°, and then at a rate which may be as high as 10 deg. per min. until room temperature is reached. The time consumed (exclusive of that required for heating and for the latter, very rapid, cooling) would be 6.5 hours.

(*b*) For the case of cooling at a rate which increases in geometrical ratio, we proceed as follows: At any moment the temporary stress,[47] $(\Delta n)_t$ is equal to that due to the momentary cooling rate less that which has already disappeared by viscous release; that is,

$$(\Delta n)_t = ch - \Delta n$$

Moreover, from equation (7c),

$$-\frac{d(\Delta n)_t}{dt} = A(\Delta n)_t^2$$

But, as already stated, the final permanent stress is equal and opposite in sign to the temporary stress which has been lost. Hence

$$\frac{d(\Delta n)}{dt} = -\frac{d(\Delta n)_t}{dt} = A(\Delta n)_t^2 = A(ch - \Delta n)^2 \qquad (11)$$

and since

$$\frac{d(\Delta n)}{dt} = \frac{d(\Delta n)}{d\theta} \cdot \frac{d\theta}{dt} = -h\frac{d(\Delta n)}{d\theta}$$

we have

$$-\frac{d(\Delta n)}{d\theta} = \frac{A}{h}(ch - \Delta n)^2 \qquad (12)$$

[47] For the sake of convenience, we use optical rather than mechanical units for the stress; *i.e.*, in the present instance we consider $\Delta n$ rather than $F$.

as an expression for the permanent stress acquired by the glass.[48] Both $A$ and $h$ are functions of $\theta$. Equation (8) may be written

$$A = A_0 2^{-\frac{\theta_0 - \theta}{p}}$$

which expresses the fact that $A$ doubles for every $p$ degrees increase in $\theta$. Obviously $p = \frac{301}{M_1}$ (see equation 8 and Table V) and is ordinarily about 10° C. For an average glass we may therefore write

$$A = A_0 2^{-\frac{\theta_0 - \theta}{10}}$$

Now, the cooling rate, $h$, is assumed to vary so that it doubles for every $q$ degrees fall in temperature, or

$$h = h_0 2^{-\frac{\theta_0 - \theta}{q}}$$

If the interval $q$ is taken as 20°, equation (12) becomes

$$-\frac{d(\triangle n)}{d\theta} = \frac{A_0}{h_0}\left\{ ch_0 - (\triangle n)\, 2^{-\frac{\theta_0 - \theta}{20}} \right\} 2^{-\frac{\theta_0 - \theta}{20}} \qquad (12a)$$

This equation can be conveniently integrated by graphical methods.[49] As an example, if the glass be annealed at a temperature given, for each kind of glass, in the fourth column of Table V, $A_0 = 0.19$; and if the initial cooling rate be taken as 6.0 deg. per hr., $h_0 = \frac{6.0}{60} = 0.10$. Moreover, for a slab 2 cm. thick $c = 13$. Substituting these values of $A_0$, $h_0$ and $c$ in equation (12a) we have

$$-\frac{d(\triangle n)}{d\theta} = \frac{d(\triangle n)}{d(\theta_0 - \theta)} = 1.9 \left\{ 1.30 - (\triangle n) \cdot 2^{-\frac{\theta_0 - \theta}{20}} \right\} 2^{-\frac{\theta_0 - \theta}{20}} \qquad (12b)$$

Figure 13 shows the curve obtained by the graphical solution of equation (12b). In this diagram the ordinates indicate the strain in $\mu\mu$ (in the middle) and the abscissæ the number of degrees through which the glass has cooled. It is to be noted that below 150° from the starting-point—that is to say, below what we have called the "annealing range"—very little additional strain

[48] It is to be noted that we neglect the fact that the glass is not uniform in temperature—an assumption which (at least for pieces of ordinary size) will not introduce any serious error.

[49] See Carl Runge: "Graphical Methods." New York, 1912, p. 120, *et seq.*

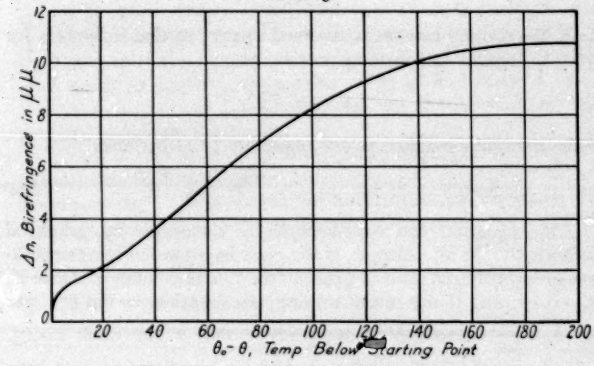
is introduced. As shown by the figure, the final strain in this case is about 10.7$\mu\mu$ per cm.

The time required for cooling is obviously

$$t_c = -\int_{\theta_0}^{\theta} \frac{d\theta}{h} = -\frac{1}{h_0}\int_{\theta_0}^{\theta} 2^{-\frac{\theta_0-\theta}{20}}\, d\theta$$

if $h$ doubles every 20°, and even if as above we make the limiting rate 10 deg. per min. (for $a = 1$), it can be shown that the total

FIG. 13.




Graphical solution of equation (12b). The abscissæ represent the number of degrees through which the glass has cooled, and the ordinates represent birefringence in $\mu\mu$.

cooling-time is approximately equal to the above integral, which if $\theta$ and $\theta_0$ differ by 100° or more, is approximately

$$t_c = \frac{29}{h_0} \qquad (13)$$

In the example given we have taken $h_0$ equal to 6.0 deg. per hr. which equals 0.10 deg. per min. Hence the cooling time is approximately $\frac{29}{0.10} = 290$ minutes, when 10.7$\mu\mu$ of strain is incurred. Now it so happens that for small values of $h_0$, the strain introduced is nearly proportional to $h_0$ and hence inversely proportional to $t_c$. If only 5$\mu\mu$ is to be incurred the initial cooling rate would be $\frac{5}{10.7} \times 6.0 = 2.8$ deg. per hr. and the cooling time would be $\frac{10.7}{5} \times 290 =$

620 minutes, or over 10 hours. Comparing this value with the corresponding one for method Ia (linear cooling), we have 10 hours as against 7 hours. It is evident, therefore, that when glass is annealed in the customary manner—that is, at a comparatively high temperature—nothing is gained by increasing the cooling rate so fast that at the *beginning* it doubles for every 20° drop in temperature. Furthermore, the higher the temperature the more disadvantageous it is to increase the cooling-rate rapidly at the start.

(II) When glass is annealed from a relatively low temperature the annealing time is large enough to be taken account of and can not be neglected as when the annealing is carried out at a high temperature. Furthermore, for the sake of generality it is necessary to consider that the glass is not *completely* annealed (as in the preceding examples) at the high temperature, but that at this temperature the stress is reduced to a certain amount $(\Delta n)_a$, the difference between this and the permissible amount being incurred during the cooling process. If $5\mu\mu$ per cm. be taken as the permissible final strain, then $5-(\Delta n)_a$ is introduced during cooling. At relatively low temperatures the temporary stress (equal to $ch'$) will ordinarily be much greater than the final permanent stress. In equation (11), therefore, $\Delta n$, being small in comparison with $ch'$, may be neglected on the right-hand side of the equation and we have for the strain introduced by cooling:

$$\frac{d(\Delta n)}{dt} = Ac^2h'^2 \qquad (14)$$

Similarly equation (12) becomes

$$-\frac{d(\Delta n)}{d\theta} = Ac^2h' \qquad (15)$$

or since $A = A_{o2}^{-\frac{\theta_o-\theta}{10}}$

$$-\frac{d(\Delta n)}{d\theta} = A_o c^2 h' 2^{-\frac{\theta_o-\theta}{10}} \qquad (16)$$

(*a*) *Cooling at constant rate.* As in case (I, a), let us assume that the glass is cooled at a constant rate, $h'$, down to the lower end of the annealing range, and thereafter at a very high

rate equal to $\frac{10}{a^2}$ deg. C. per min. From (16) the strain introduced by cooling from $\theta_0$ to $\theta$ is

$$c^2 A_0 h' \int_{\theta_0}^{\theta} 2^{-\frac{\theta_0 - \theta}{10}} \, d\theta .$$

The value of the integral, if $\theta_0 - \theta$ is at least a large fraction of 100°, is approximately 10/ln 2 = 14.5. Then, the strain to be acquired during cooling is

$$(\Delta n)_c = 5 - (\Delta n)_a = 14.5\, c^2 A_0 h' \qquad (17')$$

To find the cooling time we assume that the constant cooling extends over a range [50] of 90°. The cooling time, $t_c$ (exclusive of the time for the final very rapid cooling) is therefore

$$t_c = \frac{90}{h'} \qquad (18')$$

and substituting the value of $h'$ from (17′) we have

$$t_c = \frac{90 \times 14.5\, c^2 A_0}{5 - (\Delta n)_a} \qquad (19')$$

Moreover, from equation (7c) the annealing time is

$$t_a = \frac{\frac{1}{(\Delta n)_a} - \frac{1}{50}}{A_0} \qquad (20)$$

From these expressions for $t_c$ and $t_a$ it is important to find the relation between $A_0$, the annealing constant (and thence the annealing temperature), $(\Delta n)_a$, the degree of initial annealing, and $h_0$, the subsequent cooling rate, so that the total time required for the annealing process shall be a minimum. This total time is

$$t_a + t_c = \frac{\frac{1}{(\Delta n_a)} - \frac{1}{50}}{A_0} + \frac{90 \times 14.5\, c^2 A_0}{5 - (\Delta n)_a}$$

By partial differentiation with respect to $A_0$ and with respect to $(\Delta n)_a$ it is found that the total time is a minimum when

$$t_a = t_c \qquad (21)$$

and

$$(\Delta n)_a = 2.57 \qquad (22)$$

[50] The justification for this assumption will be apparent from what follows. In any case, the proper range can be found by trial and approximation, and moreover, a considerable variation in the assumed range will not vitiate the deductions which follow.

It is also found that

$$A_s = \frac{0.95}{c\sqrt{90 \times 14.5}} \tag{23'}$$

and

$$h_s' = \frac{90}{0.37} A_s = \frac{6.4}{c} \tag{24'}$$

Now, the validity of these relations depends on the assumption that at any moment the stresses are proportional to the rate of cooling. It is to be noted, however, that when the glass, after being annealed until the stress is $(\Delta n)_a$, is cooled at a certain rate, the first part of the cooling rate merely neutralizes the stress already present. The rate necessary to neutralize the stress $(\Delta n)_a$ is $\frac{(\Delta n)_a}{c}$, and hence at any moment the actual stress is equal to $c(h - \frac{(\Delta n)_a}{c})$. The above equations are therefore more nearly correct if $h'$ is not the total cooling rate, but is the rate in excess of that required to neutralize the residual stress. If then $h$ is the true cooling rate

$$h = h' + \frac{(\Delta n)_a}{c}$$

The correct relations may be obtained by revising the preceding equation, taking proper account of the difference between $h$ and $h'$. Making the appropriate changes [51] in the equations, we find that $h$ should be equal to 1.4 $h'$ and that as before, for minimum time

$$(\Delta n)_a = 2.57 \tag{22}$$

The equation for the strain incurred by the cooling becomes

$$(\Delta n)_c = 5 - (\Delta n)_a = 10 \times c^2 A_s (h - \frac{(\Delta n)_a}{c}) \tag{17}$$

The equation for the cooling time may now be written

$$t_c = \frac{90}{h} \tag{18}$$

or

$$t_c = \frac{64 \times 10\, c^2 A_s}{5 - (\Delta n)_a} = 264\, c^2 A_s \tag{19}$$

[51] It will be noted that the change made is in some cases a correction and in others a substitution.

while on the other hand, the equation for the annealing time remains the same:

$$t_a = \frac{\frac{1}{(\Delta n)_a} - \frac{1}{50}}{A_a} = \frac{0.37}{A_a} \qquad (20)$$

The correct value for the annealing constant is now

$$A_a = \frac{0.95 \times 1.4}{c\sqrt{90 \times 14.5}} = \frac{0.037}{c} \qquad (23)$$

and the subsequent cooling rate is

$$h = \frac{6.4 \times 1.4}{c} = \frac{9.0}{c} \qquad (24)$$

Finally, then, if glass is to be annealed at a comparatively low temperature, with subsequent cooling at a constant rate, the total time required, that is, the sum of the annealing time and the cooling time, will be least when the glass is annealed down to one-half the allowable strain by holding at a temperature at which $A_a = 0.037/c$, and then cooling at constant rate equal to $9.0/c$. deg. per min. The constant rate is continued down to the lower end of the "annealing range," after which it may be cooled at the much greater rate, $\frac{10}{a^2}$. As an example: for a plate of glass 2 cm. thick, $a = 1$, and hence $c = 13a^2 = 13$. Hence, by equation (23), $A_a = 0.037/13 = 0.00285$. For ordinary crown glass this corresponds to a temperature of 511° (see equation (8) and Table V). The cooling rate by equation (24) is $9.0/13 = 0.69$ deg. per min. $= 41$ deg. per hr. The cooling time will be approximately $\frac{90}{41} = 2.2$ hours and the total time required will be approximately twice this, or 4.4 hours.

(*b*) *Cooling at an increasing rate.* If we assume that at comparatively low temperatures the release of stress is represented by $-\mathrm{d}(\Delta n)/\mathrm{d}\theta = Ac^2h'$, it can be shown [52] that for the introduction

[52] The cooling time between any two temperatures $\theta_1$ and $\theta_2$ is obviously $t_c = \int_{\theta_1}^{\theta_2} \frac{\mathrm{d}\theta}{h}$, and the strain introduced is $(\Delta n)_c = c^2 \int_{\theta_1}^{\theta_2} Ah\mathrm{d}\theta$. To find $h$ as a function of $\theta$ so that $t_c$ and $(\Delta n)_c$ will both have minimum values, we apply the methods of the calculus of variations (*cf.* B. Williamson: "Integral Calculus," 7th ed., p. 437), which show that for a slight variation in the function defining $h$, $c^2A = b/h^3$ where $b$ is a constant. That is, $Ah^3 = \frac{b}{c^2} =$ constant.

of a given amount of strain in the cooling process, $h'$ should be progressively increased so that it doubles for every $2p$ degrees drop in temperature. Since $p$ is ordinarily about 10°, $h'$ should double for every 20°. That is, $h' = h'_o \, 2^{\frac{\theta_o - \theta}{20}}$ in which $h'_o$ is the value of $h'$ at $\theta_o$. Since $Ah'^2$ remains constant and equal to its initial value, equation (14) may be written

$$\frac{d(\triangle n)}{dt} = c^2 A h'^2 = c^2 A_o h_o'^2 = constant. \tag{25}$$

Starting with this equation and proceeding, as in IIa, we find finally that $h = h' + \frac{(\triangle n)_a}{c} = h' + h_o'$,
or $h_o = 2h_o'$. Hence,

$$h_o = \frac{h_o}{2}\left(1 + 2^{\frac{\theta_o - \theta}{20}}\right) \tag{26}$$

which expresses the fact that the cooling rate is made up of two parts, one of which is constant while the other doubles for each 20° drop in temperature. It follows that the strain incurred during cooling is

$$(\triangle n)_c = 5 - (\triangle n)_a = \frac{c^2 A_o h_o^2 t_c}{4} \tag{27}$$

and the cooling time is approximately

$$t_c = \frac{40}{h_o} \tag{28}$$

or

$$t_c = \frac{20^2 c^2 A_o}{5 - (\triangle n)_a} \tag{29}$$

As before, the annealing time is

$$t_a = \frac{\frac{1}{(\triangle n)_a} - \frac{1}{50}}{A_o} \tag{30}$$

---

Hence $h^2$ is inversely proportional to $A$, or $h^3$ doubles when $A$ is halved, and since $A$ is halved for each $p$ degrees drop in temperature, $h$ doubles for each $2p$ degrees, or about 20°. The validity of this proof is somewhat affected by the failure to distinguish between $h$, the true cooling rate, and $h'$, the cooling rate in excess of that required to neutralize the residual strain $(\Delta n)_a$. Taking account of this fact, however, we find that the requirement for minimum cooling time is approximately that $h'$, not $h$, doubles for every 20 degrees drop in temperature.

and for minimum total time $t_a = t_c$ and $(\Delta n)_a = 2.57$. Hence the glass should be annealed at the temperature at which

$$A_o = \frac{0.047}{c} \tag{31}$$

and cooled at a rate which initially is

$$h_o = \frac{5.2}{c} \tag{32}$$

and which increases progressively according to equation (26).

For a slab of glass 2 cm. thick the value of $c$ is about 13, and hence $A_o$ should be $0.047/13 = 0.0036$, which in the case of ordinary crown glass corresponds to a temperature of 514°; that is, nearly 60° below the upper limit of the annealing range. The annealing time would be $0.37/0.0036 = 103$ minutes $= 1.7$ hours, and the total time [53] for annealing plus cooling would be twice this or 3.4 hours. The initial cooling rate by equation (32) would be $\frac{5.2}{13} \times 60 = 24°$ per hr. Both $A_o$ and $h_o$ are inversely proportional to $c$, which is proportional to the square of the thickness. Hence, the thicker the glass the lower the temperature at which it is annealed and the slower is the initial cooling.

The total time required by each of the four procedures considered follows: Ia, 6.5 hrs.; Ib, 10.3 hrs.; IIa, 4.4 hrs.; and IIb, 3.4 hrs. It appears, therefore, that the low-temperature annealing procedure (methods IIa and IIb) has in addition to the other advantages enumerated above, the advantage of requiring less time. This saving of time becomes more important as the size of the glass is increased.

### ANNEALING SCHEDULES FOR VARIOUS KINDS OF GLASS.

*Optical Glass.*—On the basis of the preceding discussion it is possible to state with fair precision the best procedure for annealing glass of various kinds and of various sizes and shapes. Methods IIa and IIb are both satisfactory for optical glass. IIa (cooling at constant rate), as compared with IIb (cooling at increasing rate), requires a lower temperature but a slightly

[53] Exclusive, as before, of the short time required for the latter, very rapid cooling.

longer total time. The constant rate, according to IIa, however, is larger than the initial rate, according to IIb, a fact which tends to make IIa the more advantageous; but the rate by method IIb is increased so rapidly that this advantage holds only for the first small portion of the cooling. On the whole, we believe that method IIb is the best for the practical annealing of optical glass. The procedure recommended is therefore briefly as follows: If the glass to be annealed is cold, it is heated rapidly [54] to a temperature at which $A_\theta$, as determined from equation (8) and Table V, is equal to $0.0047/c$. If the glass is hot, as, for example, after being pressed or molded, it is brought immediately into a kiln or furnace previously heated to the given temperature. The glass is held at this temperature for $0.37/A_\theta$ minutes, and is then cooled at a rate which initially is $5.2/c$ deg. C. per min., and which is gradually increased according to the relation $h = \frac{h_0}{2}(1 + 2^{\frac{\theta_0 - \theta}{20}})$ until the rate is $10/a^2$, after which the rate is, so far as is practicable, maintained constant. The value of $c$, as already explained, depends on the size and shape of the glass blocks, but varies comparatively little for various kinds of glass. Optical glass is usually supplied in the form of plates or slabs, for which the value of $c$ is about $13a^2$, $a$ being the semi-thickness. For other shapes the value of $c$ can be estimated from equations (55) and (56). Since $c$ is proportional to $a^2$, it is obvious that doubling the thickness decreases the annealing temperature by about 20°, and increases the annealing time (and also the cooling time) to four times its original value.

In Table VII are given condensed annealing schedules for slabs of various thickness, as calculated from equations (30), (31), and (32), using the values of $A$ obtained from equation (8) and Table V. To the annealing times for the corresponding annealing temperatures have been applied a factor of safety equal to 2, in order to compensate for possible errors in temperature measurement in the kilns or furnaces in which the annealing is to be done. Thus a slab 10 cm. thick should, according to the

[54] The heating rate, if approximately uniform, may be much greater than the maximum cooling rate (see below). A heating rate of $\frac{20}{a^2}$ to $\frac{30}{a^2}$ deg. C. per min. is quite safe.

TABLE VII.

*Annealing Schedule for Optical Glass Slabs of Varying Thickness, Assuming that Strain Along the Middle (When the Glass is Viewed Through the Edges) is Finally to be no More Than 5μμ per cm.*

(Hold the glass at the proper annealing temperature for the indicated time and cool at the indicated rate.)

| Thickness | 1 cm. | 2 cm. | 5 cm. | 10 cm. | 20 cm. |
|---|---|---|---|---|---|
| Kind of glass | Annealing temperatures, deg. C. | | | | |
| Borosilicate crown | 561 | 541 | 515 | 495 | 475 |
| Ordinary crown | 535 | 514 | 487 | 466 | 445 |
| Light barium crown | 571 | 552 | 527 | 508 | 489 |
| Heavy barium crown | 608 | 592 | 572 | 556 | 540 |
| Barium flint | 516 | 494 | 466 | 454 | 423 |
| Light flint | 427 | 408 | 384 | 366 | 348 |
| Medium flint | 434 | 418 | 397 | 382 | 366 |
| Heavy flint | 424 | 407 | 386 | 370 | 353 |
| Extra heavy flint | 400 | 382 | 357 | 339 | 321 |

| | Annealing times | | | | |
|---|---|---|---|---|---|
| | 50 min. | 3½ hrs. | 21 hrs. | 86 hrs. | 14 days |
| | Cooling rates—degrees C. per hour | | | | |
| Initial rate | 96 | 24 | 3.8 | 1.0 | 0.2 |
| Rate after 10° | 116 | 29 | 4.6 | 1.2 | .3 |
| Rate after 20° | 144 | 36 | 5.8 | 1.4 | .4 |
| Rate after 30° | 184 | 46 | 7.4 | 1.8 | .5 |
| Rate after 40° | 240 | 60 | 9.6 | 2.4 | .6 |
| Rate after 50° | 319 | 80 | 13. | 3.2 | .8 |
| Rate after 60° | 432 | 108 | 17. | 4.3 | 1.1 |
| Rate after 70° | 591 | 148 | 24. | 5.9 | 1.5 |
| Rate after 80° | 816 | 204 | 33. | 8.2 | 2.0 |
| Rate after 90° | 1134 | 283 | 55. | 11. | 2.8 |
| Rate after 100° | 1584 | 396 | 63. | 16. | 4.0 |
| Maximum cooling rate | 2400 | 600 | 96. | 24. | 6. |
| Maximum heating rate | 7200 | 1800 | 288. | 72. | 18. |

formulæ, be annealed at the proper temperature for 43 hours, but the table gives twice this value, or 86 hours. That this factor of safety is unnecessary if the temperature is known to within a degree or two is proved by the following experiment: A slab of borosilicate crown glass 2 cm. thick and 7.6 cm. long, which showed when viewed lengthwise a strain (along the middle) corresponding to 850μμ; that is, $850/7.6 = 112\mu\mu$ per cm., was subjected to the thermal treatment indicated in Table VIII. The furnace was the same in which the previously described measurements were made. The total time from the moment the cold

TABLE VIII.

*Record of the Actual Annealing of a Slab of Borosilicate Glass 7.6 cm. long and 2 cm. thick. Original Stress along Middle, 850μμ, or 112μμ per cm. Total Time, 4 hr. 50 min. Strain after Annealing, 46μμ, or 6μμ per cm.*

| Time | Temp. °C. | Remarks |
|---|---|---|
| 9 : 45 A.M. | 25 | Glass put in cold furnace. |
| : 56 A.M. | 179 | |
| 10 : 08 A.M. | 337 | Temporary strain about 2000μμ. |
| : 25 A.M. | 496 | |
| : 31 A.M. | 539 | |
| : 33 A.M. | 543.3 | Temperature held approximately constant. |
| : 35 A.M. | 542.1 | |
| : 40 A.M. | 541.6 | |
| : 43 A.M. | 541.8 | |
| : 44 A.M. | 542.0 | |
| 12 : 18 P.M. | 542.8 | Cooling started. |
| : 20 P.M. | ..... | Cooling rate 0.37 deg./min. |
| : 26 P.M. | 540.7 | |
| : 28 P.M. | ..... | Cooling rate 0.40 deg./min. |
| : 35 P.M. | ..... | Cooling rate 0.48 deg./min. |
| : 38 P.M. | 535.5 | |
| : 39 P.M. | ..... | Cooling rate 0.44 deg./min. |
| : 43 P.M. | ..... | Cooling rate 0.58 deg./min. |
| : 50 P.M. | ..... | Cooling rate 0.51 deg./min. |
| : 53 P.M. | 527.7 | |
| : 55 P.M. | ..... | Cooling rate 0.54 deg./min. |
| 1 : 06 P.M. | 520.8 | |
| : 08 P.M. | ..... | Cooling rate 0.65 deg./min. |
| : 11 P.M. | ..... | Cooling rate 0.71 deg./min. |
| : 17 P.M. | 512.5 | |
| : 20 P.M. | ..... | Cooling rate 0.84 deg./min. |
| : 24 P.M. | ..... | Cooling rate 0.88 deg./min. |
| : 28 P.M. | 502.5 | |
| : 30 P.M. | ..... | Cooling rate 1.00 deg. /min. |
| : 34 P.M. | ..... | Cooling rate 1.20 deg./min. |
| : 38 P.M. | 490.3 | |
| : 40 P.M. | ..... | Cooling rate 1.35 deg./min. |
| : 41 P.M. | ..... | Cooling rate 1.5 deg./min. |
| : 42 P.M. | ..... | Cooling rate 1.7 deg./min. |
| : 44 P.M. | 480.1 | |
| : 46 P.M. | ..... | Cooling rate 2.0 deg./min. |
| : 48 P.M. | ..... | Cooling rate 2.2 deg./min. |
| : 49 P.M. | 469.0 | |
| : 51 P.M. | 359.0 | |
| : 52 P.M. | ..... | Cooling rate 3.2 deg./min. |
| : 57 P.M. | 439.0 | |
| : 58 P.M. | ..... | Door partly open. |
| 2 : 00 P.M. | ..... | Door fully open. |
| 2 : 35 P.M. | ..... | Glass taken out of furnace. |

glass was put into the furnace until it was taken out,[55] was 4 hours and 50 minutes, and the final strain was 45μμ, or about

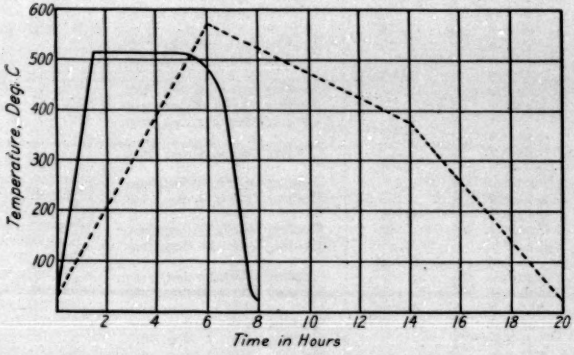
[55] The temperature of the *glass* at this moment was not measured, but was probably somewhat above 100°.

6$\mu\mu$ per cm., which—within little more than the error of experiment—was the expected amount.

In Fig. 14 the procedure now recommended for annealing optical glass is compared graphically with the hitherto customary method for the specific case of a slab of ordinary crown glass 2 cm. thick.

While the directions in Table VII are strictly applicable only when the glass is in the form of plates or slabs, and when the standard of annealing is put at 5$\mu\mu$, the necessary modification of the schedules for the case of another shape or another standard

FIG. 14.




Graphical comparison of the annealing procedure now recommended (full line) and the procedure ordinarily followed (dotted line).

of annealing can be readily made. For example, if the value of $c$ were doubled, $A_0$ would be halved, the annealing temperature would be lowered about 10°, and the annealing and cooling time would each be doubled. On the other hand, an increase in the allowable amount of strain would not affect the annealing temperature but would decrease both the annealing and the cooling time.

*Plate Glass.*—The procedure for plate glass will differ from that for optical glass mainly because the quality of annealing need not be so good. Indeed, the principal requirement for plate

glass is that after casting, it be somehow cooled down to room temperature without breaking.

It is interesting to note, moreover, that since a given amount of final permanent strain is a consequence of the release of the same amount of temporary reverse strain in the early stages of cooling, a plate of glass which is initially cooled so as to show permanent strain can be safely cooled at a correspondingly greater rate than a plate which finally shows no strain. In this respect, therefore, a certain amount of strain is an advantage, but on the other hand, an excess of strain, as explained below, would render the glass difficult to cut and would make it less resistant to sudden *rises* in temperature.

There seems to be no information available on which to estimate the limiting strain which could be allowed in plate glass. Ordinary plate glass, however, commonly shows 30$\mu\mu$ per cm. (along the middle when viewed longitudinally), and it is probably safe to assume that nothing would be gained by making the strain any less than that amount.

Twyman's [56] proposal that the internal stresses in commercial glassware should be limited to 1/20 of the heating stress would correspond to about 100$\mu\mu$ per cm. In the absence of more definite information on the subject, we may take 50$\mu\mu$ per cm. as the desirable upper limit of strain in plate glass.

The composition of plate glass is commonly about the same as that of the ordinary crown glass which we have used. In determining a suitable annealing schedule the annealing constants for ordinary crown glass may therefore be used. For the sake of simplicity we would choose method IIa (see above), and by making the appropriate changes in equations (17) to (24) so as to take account of the fact that the glass is to be annealed down to 25$\mu\mu$ per cm. by holding at a given temperature, and that the additional 25$\mu\mu$ strain is to be acquired during cooling, we arrive at the conclusion that plate glass should be annealed by holding at such a temperature that $A = \frac{0.037}{c}$, and then cooling at a constant rate equal to $\frac{80}{c}$ = for the first 100°. Thus for sheets 2 cm. thick ($c = 13$) the annealing process would be as follows: After being

[56] *Loc. cit.*

cast, the glass would be brought to a temperature of about 511°, held at that temperature for 15 minutes, and then cooled at a constant rate equal to 6° C. per minute until the temperature had fallen 100°, after which (see above) the cooling rate may safely be as high as 10 or 20 degrees per minute. For thicker or thinner plates the annealing temperature would vary, and the annealing and cooling time would be correspondingly increased or decreased.

*Other Kinds of Glass.*—For the various other kinds of glass articles such as bottles, lamp-bulbs, cut-glass, chemical glassware, and so on, much that has been said in connection with the annealing of plate glass will apply. The problem is complicated, however, by the variety of compositions and of shapes which are encountered. In general, before an annealing schedule can be given for any one kind of glass, its annealing characteristics must be determined experimentally by measurements at two or more temperatures, but since the change with temperature of the annealing rate is roughly the same for all glasses and great precision is not necessary, a knowledge of the annealing rate at one temperature or of the "annealing temperature" for some specified set of conditions, will usually suffice. The "annealing temperatures" as given by English and Turner (see Table VI) for various glasses, afford useful information from which, by comparison with the results we have given, an approximately correct annealing procedure for many of the glasses can be deduced. As with plate glass, a limiting strain of 50$\mu\mu$ per cm. is recommended, and the complication due to varied shapes may in many cases be circumvented by treating the article (*e.g.*, a bottle, or a beaker) as a slab of thickness equal to twice the maximum thickness of any part of the given article.

### TEMPERATURE DISTRIBUTION AND STRESSES: OPTICAL PATH DIFFERENCES DUE TO THERMAL GRADIENTS.

*Temperature Distribution During Heating and Cooling.*—The first step in the estimation of the thermal stresses produced in glass is the calculation of the temperature gradients under the assumed conditions. Appropriate formulæ for two types of heating have already been given.[57]

[57] E. D. Williamson and L. H. Adams: *Phys. Rev.*, **14**, 99–114 (1919).

For the case of the *sudden change of surface-temperature* the three most generally useful [58] formulæ are:

*Slab*

$$H = 2 \sum_{m=1}^{m=\infty} \left\{ \frac{1}{Q_m (-1)^{m+1}} e^{-\frac{\kappa Q^2_m t}{a^2}} \cos\left(\frac{Q_m x}{a}\right) \right\} \qquad (33)$$

*Cylindrical rod*

$$H = 2 \sum_{m=1}^{m=\infty} \left\{ \frac{1}{R_m J_1(R_m)} e^{-\frac{\kappa R^2_m t}{a^2}} J_0\left(\frac{R_m x}{a}\right) \right\} \qquad (34)$$

*Sphere*

$$H = 2 \sum_{m=1}^{m=\infty} \left\{ \frac{1}{S_m (-1)^{m+1}} e^{-\frac{\kappa S^2_m t}{a^2}} \frac{a}{x} \sin\left(\frac{S_m x}{a}\right) \right\} \qquad (35)$$

where

$$H = \frac{\theta - \theta_f}{\theta_i - \theta_f}$$

$\theta$ is the temperature at any point; $\theta_i$, the initial uniform temperature of the solid; $\theta_f$, the constant temperature of the surface; $\kappa$, the thermal diffusivity (equal to the thermal conductivity divided by the product of the specific heat and the density); $t$, the time;

[58] For points close to the surface or for small values of $t$ the infinite series (33), (34), and (35) are not very rapidly convergent. Under these conditions the following formulæ are more convenient:

*Sphere*

$$1 - H = \frac{2a}{x\sqrt{\pi}} \left\{ \int_{q_0 - q}^{q_0 + q} e^{-z^2} dz + \int_{3q_0 - q}^{3q_0 + q} e^{-z^2} dz + \int_{5q_0 - q}^{5q_0 + q} e^{-z^2} dz + \ldots\ldots \right\}$$

*Slab*

$$1 - H = \frac{2}{\sqrt{\pi}} \left\{ \int_{q_0 - q}^{3q_0 + q} e^{-z^2} dz + \int_{q_0 + q}^{3q_0 - q} e^{-z^2} dz + \int_{5q_0 - q}^{7q_0 + q} e^{-z^2} dz + \int_{5q_0 + q}^{7q_0 - q} e^{-z^2} dz + \ldots\ldots \right\}$$

*or*

$$H = \frac{2}{\sqrt{\pi}} \left\{ \int_{0}^{q_0 - q} e^{-z^2} dz - \int_{q_0 + q}^{3q_0 - q} e^{-z^2} dz + \int_{3q_0 + q}^{5q_0 - q} e^{-z^2} dz - \ldots\ldots \right\}$$

where $z$ is an integration variable, $q = x/2\sqrt{\kappa t}$, $q_0 = a/2\sqrt{\kappa t}$ and $H$, $a$, $x$, $K$ and $t$ have the same meaning as above. *Cf.* R. S. Woodward: *Phys. Rev.*, **16**, 176, 177 (1903); and A. L. Queneau: *School of Mines Quart.*, **23**, 181–195 (1902).

$a$, one-half the diameter or thickness; $x$, the distance from the centre; $J_0$ ( ) and $J_1$ ( ) are Bessel's functions of the zeroth and first order; $m$ is any integer; and $Q_m$, $S_m$, and $R_m$ are written respectively for $(2m-1)\pi/2$, $m\pi$, and the roots of the equation $J_0(x)=0$.

For the case of *linear heating* of the surface, we have the corresponding equations:

*Slab*

$$\theta = ht - \frac{h}{2\kappa}(a^2 - x^2) + \frac{2ha^2}{\kappa}\sum_{m=1}^{m=\infty} \frac{1}{Q_m^3(-1)^{m+1}} e^{-\frac{\kappa Q_m^2 t}{a^2}} \cos Q_m\left(\frac{x}{a}\right). \quad (36)$$

*Cylinder*

$$\theta = ht - \frac{h}{4\kappa}(a^2 - x^2) + \frac{2ha^2}{\kappa}\sum_{m=1}^{m=\infty} \frac{1}{R_m^3 J_1(R_m)} e^{-\frac{\kappa R_m^2 t}{a^2}} J_0\left(R_m\frac{x}{a}\right). \quad (37)$$

*Sphere*

$$\theta = ht - \frac{h}{6\kappa}(a^2 - x^2) + \frac{2ha^2}{\kappa}\sum_{m=1}^{m=\infty} \frac{1}{S_m^3(-1)^{m+1}} e^{-\frac{\kappa S_m^2 t}{a^2}} \frac{a}{x}\sin\left(S_m\frac{x}{a}\right). \quad (38)$$

When $t$ became large with respect to $a$, the last term of (36), (37), or (38) vanishes, and the temperature gradient reaches a "steady state" in which the temperature distribution is represented by a parabola. The equations for this steady state are:

*Slab*

$$\theta = ht - \frac{h}{2\kappa}(a^2 - x^2) \quad (39)$$

*Cylinder*

$$\theta = ht - \frac{h}{4\kappa}(a^2 - x^2) \quad (40)$$

*Sphere*

$$\theta = ht - \frac{h}{6\kappa}(a^2 - x^2) \quad (41)$$

Finally for *linear heating* of cylindrical tubes and spherical shells, if $a_1$ and $a_2$ are respectively the internal and external radii, we have the steady state equations.[89]

*Cylindrical Tube.*—(a) Heat transfer taking place only at outside,

$$\theta = ht - \frac{h}{4\kappa}\left(a_2^2 - x^2\right) + \frac{ha_1^2}{2\kappa}\ln\frac{a_2}{x} \quad (42a)$$

[89] The general equations corresponding to (42a), (42b), and (43) can be obtained by the proper substitutions.

(*b*) Heat transfer taking place equally at inside and outside,

$$\theta = ht + \frac{hx^2}{4\kappa} + \frac{h\left(a_1^2 \log \frac{x}{a_2} - a_2^2 \log \frac{x}{a_1}\right)}{4\kappa \log \frac{a_2}{a_1}} \qquad (42b)$$

*Spherical Shell.*—Heat transfer taking place only at outside,

$$\theta = ht + \frac{h(x^2 - a_2^2)}{6\kappa} + \frac{ha_1^3}{3\kappa}\left(\frac{1}{x} - \frac{1}{a_2}\right) \qquad (43)$$

*Stresses Due to Temperature Gradients.*—These stresses may be evaluated from the following equations:[60] in which $\alpha$ is the coefficient of linear (thermal) expansion, $e = \frac{1}{3K} + \frac{1}{9K}$, $f = \frac{1}{6K} - \frac{1}{9K}$ and $\theta'$ is the difference between the temperature at any point and the temperature of the surface.

*Slab*

$$F = \frac{C_1 - a\theta'}{e - f} \qquad (44)$$

where

$$C_1 = \frac{a}{a} \int_0^a \theta' dx \qquad (44a)$$

and $F$ is the tension in any direction parallel to the faces of the slab.

*Cylindrical Tube.*—If the temperature distribution be symmetrical about the axis, $F_a$, $F_r$, and $F_t$, the axial, radial and tangential stresses, may be obtained from the three equations:

$$(e - f)F_a + a\theta' = C_1 \qquad (45a)$$

$$xF_r^2 = -\frac{a}{e - f} \int x\theta' dx + \frac{C_2 x^2}{2} + C_3 \qquad (45b)$$

$$F_t = F_r + x \frac{dP_r}{dx} \qquad (45c)$$

$C_1$, $C_2$ and $C_3$ being constants of integration which are evaluated from the boundary conditions. For a solid cylinder the same equations apply, except that $C_3$ is zero.

[60] *Cf.* Williamson: *loc. cit.*

*Spherical Shell.*[61]—With temperature distribution symmetrical about the centre, we have

$$x^3 F_r = -\frac{2a}{e-f}\int x^2\theta^1 dx + C_1\frac{x^3}{3} + C_2 \qquad (46a)$$

$$F_t = F_r + \frac{x}{2}\frac{dF_r}{dx} \qquad (46b)$$

$C_1$ and $C_2$ being integration constants. For a solid sphere $C_2$ vanishes.

The above equations may be used to determine the stresses in the various solids when the heating or cooling is carried out in a certain specified manner. As an example of their application, we may consider (A) linear heating of the surface, and (B) sudden change of surface temperature.

*(A) Surface Heated at Constant Rate. (Steady State.)*

(1) *Slab.*—From equations (39), together with (44) and (44a)

$$F = \frac{a\,h}{6\kappa\,(e-f)}(a^2 - 3x^2) \qquad (47)$$

*(2a) Cylindrical tube, heated on outside only.*—From equations (42a), together with (45a), (45b), and (45c)

$$F_a = \frac{a\,h\,(a_2^2 - x^2)}{4\kappa\,(e-f)} + \frac{aha_1^2}{4\kappa\,(e-f)}\left(\ln\frac{x^2}{a_2^2} - 1\right) \qquad (48)$$

$$F_r = -\frac{ahx^2}{16\kappa\,(e-f)} + \frac{aha_1^2}{4\kappa\,(e-f)}\left(\ln x - \frac{1}{2}\right) + \frac{C_1}{2} + \frac{C_2}{x^2} \qquad (48b)$$

$$F_t = -\frac{3ahx^2}{16\kappa\,(e-f)} + \frac{aha_1^2}{4\kappa\,(e-f)}\left(\ln x + \frac{1}{2}\right) + \frac{C_1}{2} - \frac{C_2}{x_2} \qquad (48c)$$

$C_1$ and $C_2$ can be immediately evaluated by placing $F_r = 0$ for $x = a_1$ and $x = a_2$.

[61] While the equations for the sphere are exact, those for the slab and cylinder involve the assumption that end-effects can be neglected, but when the thickness or diameter is small in comparison with the other dimensions, the error introduced by this assumption is probably very small.

(*2b*) *Limiting case for small bored tube* ($a_1$ *negligible compared to* $a_2$).

$$F_a = \frac{ah\,(a_2^2 - 2x^2)}{8\kappa\,(e-f)} \tag{49a}$$

$$F_r = \frac{ah\left(a_2^2 - x^2 - \frac{a_2^2 a_1^2}{x^2}\right)}{16\,\kappa\,(e-f)} \tag{49b}$$

$$F_t = \frac{ah\left(a_2^2 - 3x^2 + \frac{a_2^2 a_1^2}{x^2}\right)}{16\,\kappa\,(e-f)} \tag{49c}$$

(*2c*) *Solid Cylinder.*—From equations (40), together with (45a), (45b), and (45c), $C_3$ being zero,

$$F_a = \frac{ah\,(a^2 - 2x^2)}{8\kappa\,(e-f)} \tag{50a}$$

$$F_r = \frac{ah\,(a^2 - x^2)}{16\,\kappa\,(e-f)} \tag{50b}$$

$$F_t = \frac{ah\,(a^2 - 3x^2)}{16\kappa\,(e-f)} \tag{50c}$$

(*3a*) *Spherical Shell, Heated on Outside Surface Only.*—From equations (43), together with (46a) and (46b), putting $F_r = 0$ at $x = a_1$, and $x = a_2$,

$$F_r = \frac{ah}{15\,\kappa\,(e-f)}\left\{-x^2 - \frac{5a_1^3}{x} + \frac{a_2^5 + 5a_2^3 a_1^3 - 6a_1^5}{a_2^3 - a_1^3} - \frac{a_2^5 a_1^3 - 6a_2^3 a_1^5 + 5a_2^2 a_1^6}{(a_2^3 - a_1^3)\,x^3}\right\} \tag{51a}$$

$$F_t = \frac{ah}{15\,\kappa\,(e-f)}\left\{-2x^2 - \frac{5a_1^3}{2x} + \frac{a_2^5 + 5a_2^3 a_1^3 - 6a_1^5}{a_2^3 - a_1^3} - \frac{a_2^5 a_1^3 - 6a_2^3 a_1^5 + 5a_2^2 a_1^6}{2\,(a_2^3 - a_1^3)\,x^3}\right\} \tag{51b}$$

(*3b*) *Limiting case for very small internal diameter* ($a_1$ *negligible compared to* $a_2$).

$$F_r = \frac{ah}{15\kappa\,(e-f)}\left(-x^2 + a_2^3 - \frac{a_2^3\,a_1^3}{x^3}\right) \tag{52a}$$

$$F_t = \frac{ah}{15\kappa\,(e-f)}\left(-2x^2 + a_2^3 + \frac{a_2^3\,a_1^3}{x^3}\right) \tag{52b}$$

(3c) *Solid Sphere.*—From equation (41), together with (46a) and (46b), $C_2$ being zero,

$$F_r = \frac{a\,h}{15\kappa\,(e-f)}\,(a^2 - x^2) \tag{53a}$$

$$F_t = \frac{a\,h}{15\kappa\,(e-f)}\,(a^2 - 2x^2) \tag{53b}$$

*Approximate Equations for Glass (Linear Heating).*—For most glasses $\alpha$ is about $7.9 \times 10^{-6}$ per deg. C, $\kappa$ is about 0.004 cm.²/sec., or 0.24 cm.²/min., and from the values of $K$ and $R$ (see Table I) $e-f\left(=\frac{1}{6R}+\frac{2}{9K}\right)$ is about $1.2 \times 10^{-6}$ cm.²/kg. Hence, if $F$ measured in kg/cm.² and $h$ deg./min. (being positive for heating), we have the following approximate equations (tension being reckoned as positive and compression as negative).

*Slab*

$$F = 4.6h(a^2 - 3x^2) \tag{54}$$

At the middle, $x = 0$, and therefore

$$F = 4.6ha^2 \tag{54a}$$

while at the surface, $(x = a)$

$$F = -9.2ha^2 \tag{54b}$$

*Solid cylinder*

$$F_a = 3.4h(a^2 - 2x^2) \tag{55a}$$

$$F_r = 1.7h(a^2 - x^2) \tag{55b}$$

$$F_t = 1.7h(a^2 - 3x^2) \tag{55c}$$

*Solid sphere*

$$F_r = 1.8h(a^2 - x^2) \tag{56a}$$

$$F_t = 1.8h(a^2 - 2x^2) \tag{56b}$$

(B) *Sudden Change of Surface Temperature.*

In this case the application of the proper temperature distribution equations to the general stress equations yields, as already shown for the case of linear heating, the desired equation for the sudden change of surface temperature. Only one such equation will be given here; namely, that for the infinite slab. The combination of equations (33) and (44) leads to the relation:

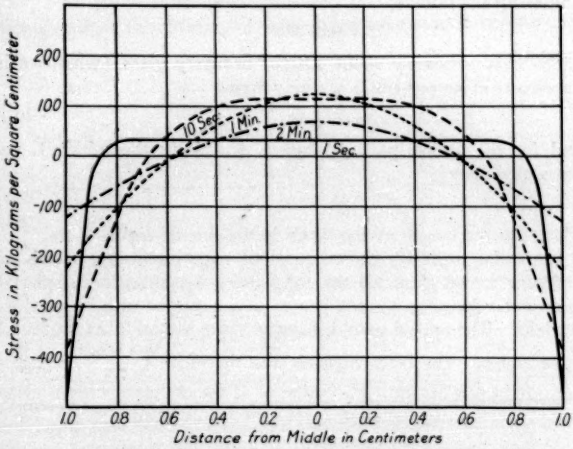
*Slab*

$$F = \frac{2\alpha(\theta_i - \theta_f)}{C} \sum e^{\frac{-\kappa Q_m^2 t}{a^2}} \left\{ \frac{1}{Q_m^2} - \frac{(-1)^{m+1}}{Q_m} \cos \frac{Q_m x}{a} \right\} \tag{57}$$

in which $C = \frac{1}{6R} + \frac{2}{9K}$, $K$ being the modulus of compressibility, $R$ the modulus of rigidity. The remaining symbols have the meanings already explained.

In Fig. 15 the distribution of stress in a slab 2 cm. thick, initially at 100° and suddenly plunged into a bath at 0°, is shown for several elapsed times. The stress distribution was calculated from equation (57) and the numerical values of $a$, $\kappa$, $K$ and $R$ were taken as the approximate average for various kinds of glass.

FIG. 15.




Curves showing the distribution of stress in a plate of glass 2 cm. thick which, initially at zero, is suddenly plunged into a bath at 100° C. The full line indicates the stresses throughout the glass after 1 second has elapsed; the broken line after 10 seconds; the dotted line after 1 minute; and the dot-dash line, after 2 minutes. These curves are computed from equation (57).

### *Optical Path Differences.*

From the stress equations already given, the optical path differences as observed between crossed nicols may be readily calculated.

Some examples will be given for the case of temporary stresses due to linear heating. These formulæ also give a close approximation to the distribution of birefringence in strained glass objects such as are ordinarily encountered.

*Slab.*—When a slab or plate is viewed lengthwise, the longitudinal stresses in the line of sight produce no birefringence and (except near the edges) there are no transverse stresses. Hence

the observed path difference at any point is due only to the single set of longitudinal stresses normal to the line of sight, that is, parallel to the faces of the plate and perpendicular to direction of the beam of light. The stress is given by equation (54) and the birefringence is $B$ times the stress at any point. Hence,

$$\triangle n = 4.6\, Bh\,(a^2 - 2x^2) \tag{58}$$

and along the middle,

$$\triangle n = 4.6\, Bha^2 \tag{59}$$

Since $B$ is ordinarily about 2.8 for ordinary glass (when $\Delta n$ is expressed in $\mu\mu$ per cm.), we may write

$$\triangle n = 13\, ha^2 \tag{60a}$$

and for the optical path difference, $\delta$, (along the middle), we have obviously

$$\delta = 13\, ha^2 l \tag{60b}$$

where $l$ is the length of the block in the line of sight.

In order to verify these equations experimentally, a slab of ordinary crown glass 7.6 cm. long and 2.2 cm. thick was placed in an oven (see Fig. 5) and heated at a constant rate of 3.8 deg. per min. The optical path difference along the middle was found to be $402\mu\mu$. The birefringence was therefore $\frac{402 \times 10^{-7}}{7.6} = 52.9 \times 10^{-7}$, or $52.9\mu\mu$ per cm. The value of $B$ for ordinary crown glass is 2.57. Hence, by equation (60a) the birefringence should have been $4.6 \times 2.57 \times 3.8 \times (\frac{2.2}{2})^2 = 54.3\mu\mu$ per cm. Another experiment with a different heating rate gave 60.2 and 60.9 for the birefringence calculated, respectively, from the heating rate and from the observed path difference.

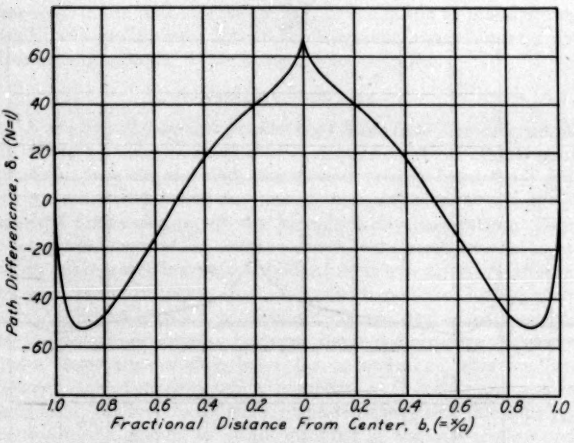
*Cylinder.*—When a cylinder is viewed along its axis, the calculation of the birefringence distribution is simple. In this case the axial stresses produce no effect, and the path difference is due to difference of the radial and tangential stresses. Hence, from equations (55b) and (55c)

$$\triangle n = 3.4\, B h x^2 \tag{61}$$

The sign of the birefringence is such that for a cylinder which is being heated or for a cylinder which is permanently strained, $\gamma$ (the greater index) is in the radial direction.

As an example of the use of equation (61) we cite the following observation: A section of a cylindrical glass rod 2.0 cm. in diameter, when viewed lengthwise, showed at the surface a birefringence equal to $2.1 \times 10^{-4}$ (2100$\mu\mu$ per cm.). The rod was of soda lime glass of unknown composition, but it is safe to assume that the value of $B$ is not far from 2.8. Hence, by equation (61) the value of $h$ is $\frac{2100}{2.8 \times 34 \times 1^2} = 220$ deg. per min. That is, after being drawn this glass rod must have been cooled through the

FIG. 16.




This, the graph of equation (62), shows the optical path difference, $\delta$, when a beam of light passes through a strained cylindrical rod in a direction normal to the axis. Reflection of light at the surface is supposed to be prevented by immersing the rod in a liquid of the same refractive index.

annealing range at a rate of about 200 deg. C. per minute. It is of interest to note that, according to equation (55c) the stress at the surface of this glass was $3.4 \times 220 = 750$ kg/cm.$^2$

When the cylinder is viewed in a direction normal to the axis the problem is much more complicated. In this case the observed path difference is due to the axial stress and to the components (in a direction normal to the line of sight and to the axis) of

the radial and tangential stresses. Performing the necessary substitutions and integrating along a chord, we find that

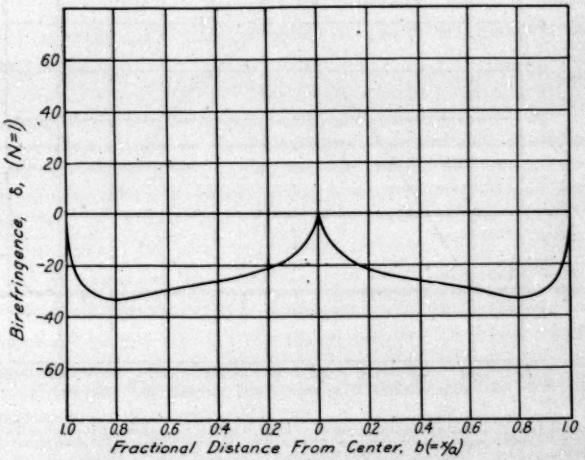
$$\frac{\delta}{N} = b - b^3 + \sqrt{1-b^2}\left(\frac{2}{3} + \frac{b}{2} - \frac{8b^2}{3}\right) + \left(-\frac{b}{2} + \frac{b^3}{4}\right) \ln \frac{1+\sqrt{1-b^2}}{1-\sqrt{1-b^2}} \quad (62)$$

in which $\delta$ is the path difference along a chord whose distance from the centre is $x$; $b$ is equal to $x/a$, $a$ being the radius; and

$$N = \frac{a k B a^3}{8 (e-f) \kappa}$$

In Fig. 16 is shown the graph of equation (62). A qualitative check on the validity of this equation was obtained by placing

FIG. 17.




Curve (graph of equation 63) showing optical path difference, $\delta$, when a beam of light passes through a strained sphere. The value of $N$ (as also in Fig. 16) is taken as unity.

a strained glass rod in a square glass jar, surrounding the rod with liquid of the same refractive index and viewing it between crossed nicols. The sign of the path difference is such that near the middle of the strained rod $\alpha$ is normal to the axis.

*Sphere.*—The path differences in this case are due to both the radial and the tangential stresses, and the equation which re-

sembles that for the cylinder viewed normal to the axis, is as follows:

$$\frac{\delta}{N} = -\frac{1}{3} + b - \frac{2}{3}b^3 + \sqrt{1-b^2}\left(\frac{1}{3} + \frac{b}{2} - \frac{4b^2}{3}\right) + \left(-\frac{b}{2} + \frac{b^3}{4}\right)\ln\frac{1+\sqrt{1-b^2}}{1-\sqrt{1-b^2}} \qquad (63)$$

in which

$$N = \frac{2\alpha\, hBa^3}{15(e-f)\kappa}$$

the remaining symbols having the meanings already explained. $\gamma$ is always in the radial direction. Fig. 17 is a graph of this equation which was qualitatively verified by observing Prince Rupert drops immersed in a liquid of the same refractive index.

### CONCLUDING REMARKS.

*Variation of Physical Constants of Glass with Temperature.*— It should be borne in mind that in dealing with the temperature gradients in glass and with the stresses arising from these gradients, we have assumed that the expansion coefficient, $\alpha$, and the thermal diffusivity, $\kappa$, do not change with temperature. This assumption was made in order to simplify the mathematical treatment, but it is well known that both of these quantities are affected by temperature changes. Indeed, as mentioned above, the expansion coefficient shows a pronounced change near the upper end of the annealing range. Between room temperature, however, and a temperature slightly below the upper end of what we have defined as the annealing range, it appears as if both $\alpha$ and $\kappa$ were sufficiently constant to enable our equations to give a fair approximation to the amount of strain incurred by various cooling procedures. The experiment (described above, p. 841) in which a slab of glass, when cooled at an approximately uniform rate through the annealing range showed the expected amount of permanent strain, would seem to be good presumptive evidence that, at least for practical purposes, our assumption of the constancy of $\alpha$ and $\kappa$ leads to sufficiently correct equations. The statement of Peters and Cragoe [62]: "It seems probable that most of the strains that exist in a piece of glass are introduced when it cools through the critical region," is not in accord with our ex-

[62] *Loc. cit.*, p. 142.

perience. In fact, anyone who will take the trouble to cool a piece of glass at a uniform rate from a temperature above the softening point, observing at the same time the strain in the glass, can satisfy himself that no important amount of strain is introduced in the critical range, so long as the cooling rate remains constant. Whenever the cooling rate decreases, however, the appearance of strain may be noticed.

That the sudden change in expansion coefficient should not be instrumental in introducing strain in the critical region seems to us to be due to the fact that it occurs at a relatively high temperature, so that the strains momentarily introduced by the change in $\alpha$ are relieved at a rapid rate. For example, from the results of Peters and Cragoe it appears that with light barium crown glass No. 11 (nearly identical with our light barium crown), comparatively little change in expansion coefficient takes place below 560°. Now if a piece of this glass were to cool down to 560°, acquiring strain as it cooled, the greater part of this strain would disappear in a few minutes, for even at 560° a large stress would be reduced to $10\mu\mu$ per cm. in less than 15 minutes. It is evident, therefore, that except for very rapid cooling, which would entail large temperature differences between the inside and the outside of the glass, the sudden change in the expansion coefficient is not likely of itself to introduce much strain, nor, in our opinion, to destroy the utility of the temperature and stress relations which we have recorded in this paper.

*Ability of Glass to Withstand Changes in Temperature.*—An important part of the technic of annealing glass consists in heating and cooling the glass in the shortest possible time. Now, it can be seen by inspection of equations (54) *et seq.*, that in general, when a piece of glass changes temperature, the maximum stresses occur at the surface. Furthermore, their direction is parallel to the surface, *i.e.*, they are tangential or longitudinal stresses. Thus, at the surface of a sphere when heating there is tangential compression, and when cooling, tangential tension. It is well known that the strength of glass under tension is much less than that under compression. The tensile strength of glass is seldom above 1000 kg/cm.$^2$, the average being about 600 kg/cm.$^2$, while the compressive strength may be several thousand kg/cm.$^2$. It follows, therefore, that glass articles should withstand much more violent thermal increases than decreases. This

conclusion is in accord with the well-known fact that glass cracks more easily when cooled than when heated. In the practical annealing of glass it is not often that glass is fractured by too rapid heating; indeed, it would be difficult, if not impossible, to heat the ordinary annealing furnace or muffle too rapidly; while, on the other hand, the cooling rate may easily become excessive, and should be regulated carefully. As already explained, in order to allow a fair factor of safety optical glass slabs should never be cooled at a rate greater than $10/a^2$ deg. C. per minute, $a$ being the semi-thickness of the slab.

The permanent stresses occurring in an unannealed piece of glass are qualitatively the same as the temporary stresses produced by heating, *i.e.*, longitudinal or tangential compressions at the surfaces. This means that when cutting strained glass, enough extra tension must be applied at the crack or cut to neutralize the compressive force and then to exceed the tensile strengh of the glass. Highly strained glass would therefore be difficult to cut up into pieces (as, for example, with a diamond). As illustrative of this fact, all attempts to cut a glass rod showing strain to the extent of 2000$\mu\mu$ per cm. completely failed, and it could be broken only by blows from a heavy hammer.

## SUMMARY.

The manner in which temporary and permanent stresses arise in glass is discussed. It is shown that the process of annealing glass can best be carried out if we know for the various glasses and for the various temperatures the rate of release of the internal stresses. Such measurements for nine kinds of glass are here presented. The release of stress at constant temperature was found to proceed usually according to the equation

$$\frac{1}{F} - \frac{1}{F_0} = At$$

in which $F$ is the stress at any time, $t$, $F_0$ is the initial stress, and $A$ is a constant for the particular glass at a particular temperature, and is a measure of the rate at which stresses are relieved. The variation of this rate with temperature follows the equation

$$\log A = M_1\theta - M_2$$

in which $M_1$ and $M_2$ are constants for a particular glass.

At any temperature, a glass requires a certain *annealing time*. This is arbitrarily defined as the time required to reduce the stress

(in optical units) from 50 to 2.5$\mu\mu$ per cm. For convenience of reference, the 150° interval of temperature lying immediately below the temperature at which the annealing time is 2 minutes is called the *annealing range*. At temperatures below the annealing range, as thus defined, very little permanent stress can be introduced.

Concrete directions are given for annealing various kinds of glass. It is shown that the best method for annealing requires that the glass be held at constant temperature (below the customary annealing point) for the appropriate time and then cooled at an increasing rate.

Finally, there are presented a number of equations which are convenient for calculating the stresses due to heating or cooling various shapes of glass.

GEOPHYSICAL LABORATORY,
CARNEGIE INSTITUTION OF WASHINGTON,
WASHINGTON, D. C.,
September 1, 1920.

---

**Desert Watering Places.** (*U. S. Geological Survey Press Bulletin No. 459*, November, 1920.)—Every year the "American Desert" takes its toll of human life. In many parts of it springs, wells, and water holes are few and far between. Three years ago the United States Geological Survey, Department of the Interior, under a small special appropriation by Congress, erected signposts in the worst portions of our southwestern deserts, giving directions and distances from watering place to watering place. This life-saving work of the Geological Survey was supplemented by a survey of these deserts in coöperation with the California Department of Engineering, the results of which are being published in a series of guidebooks. A guide just issued, entitled "Routes to Desert Watering Places in the Salton Sea Region, California," by J. S. Brown, contains maps of roads and watering places and road logs giving mileage and notes as to supplies obtainable.

**Congress for the Management of the Streams in the Garonne Basin.**—*Le Génie Civil* in the issue of July 31st announces that this meeting will be held at Bordeaux in 1921 under the presidency of Senator Claveille, former Minister of Public Works. The discussions will relate to the employment of streams for the development of power, for navigation, and for irrigation. The extensive project already adopted for the Rhone valley has, no doubt, been an incentive to the people living in the territory draining into the Bay of Biscay.

G. F. S.

# A PRODUCTION OF PANCHROMATIC SENSITIVENESS WITHOUT DYES.*

BY

**J. G. CAPSTAFF and E. R. BULLOCK.**

Eastman Kodak Company.

In the course of some experimental work carried out by one of the writers in 1917 with ordinary plates in which the plates were being bathed in certain solutions prior to exposure, it occasionally happened that the emulsions fogged badly on development. The cause of the fogging was puzzling, because the plates that showed the defect had apparently had the same treatment as those that developed clean. Investigation showed that the treatment undergone by the plates had rendered them panchromatic and that their erratic behavior regarding fog was mainly due to the time they had been exposed to the red light of the dark room which had been considered safe with the type of emulsion being experimented with before receiving the treatment referred to.

By a process of elimination it was finally discovered that the color sensitizing was caused by the use of a sodium bisulphite bath followed by a lengthy washing. As the phenomenon was thought to be new and to be of theoretical interest, work was commenced to determine the conditions which conferred the maximum color sensitiveness and, if possible, to explain the effect.

Considerable work has from time to time been devoted to the problem, but to date no entirely satisfactory hypothesis has been evolved, the most plausible being that the treatment results in a partial reduction of the silver salt to colloidal silver, which is known under certain conditions to act as an optical sensitizer. The work is being continued, and a detailed account of the experiments will be given later. It is thought, however, that a brief summary of the most fruitful experiments carried out so far will be of interest.

The effect is not peculiar to any one type of emulsion, but most of the trials were made on Eastman Commercial film. Fig. 1

* Communicated by Dr. C. E. K. Mees, Director. Communication No. 92 from the Research Laboratory of the Eastman Kodak Company.

is a wedge spectrograph showing the normal spectral sensitiveness of this emulsion.

In one experiment six pieces of film were bathed in 2 per cent. sodium bisulphite for ten mintes, the temperature being 70° F. They were then washed in running tap water (temperature about 60° F.) for the following times:

| | | | |
|---|---|---|---|
| 1. | Washed ..... | 5 | minutes. |
| 2. | Washed ..... | 15 | minutes. |
| 3. | Washed ..... | 1 | hour. |
| 4. | Washed ..... | 5½ | hours. |
| 5. | Washed ..... | 24 | hours. |
| 6. | Washed ..... | 30 | hours. |

All were allowed to dry spontaneously hanging in a well-ventilated dark room. The temperature averaged 70° F.

Wedge spectrograms showed progressively increasing color sensitiveness with washing time. Figs. 2 and 3 illustrate the first and last of this series.

The fog density increased with the color sensitiveness, varying, with normal development, from .07 in number one to .41 in number six.

A similar series washed in distilled water showed only the normal sensitiveness and washing in tap water without the preliminary bisulphite treatment gave a degree of color sensitiveness so slight that its existence was overlooked in the earliest experiments but which was subsequently found to be invariably produced on prolonged washing. Soaking in a large volume of water or solution (*e.g.*, 2 litres for one 3¼ x 4¼ film) was always found to have the same effect as washing in running water, provided that in each case the temperature was the same. Tap water boiled down to various extents and then cooled was found to have very much the same effect as untreated tap water.

The addition of alkali to the tap water reduced the time of washing necessary and increased the general sensitiveness. The film of spectrogram number four had the following treatment:

| | | |
|---|---|---|
| Sodium bisulphite 5 per cent. ..... | 10 | minutes. |
| Washing ..... | 5 | minutes. |
| *N*/10 sodium carbonate ..... | 10 | minutes. |
| Washing ..... | 5 | minutes. |

It was later found, however, that even better results were produced by a 0.2 per cent. distilled-water solution of

pure potassium bicarbonate which was neutral to litmus and free from chlorides.

Sodium sulphite as the preliminary bath apparently has no

FIG. 1.

FIG. 2.

FIG. 3.

FIG. 4.

action. Sulphurous acid has. It seems then that the effect is due to the sulphurous acid content of the sodium bisulphite solu-

tion. This is confirmed by the fact that the addition of a little sulphuric acid to the bisulphite accentuates the action of the bath.

Extremely small percentages of soluble bromides or chlorides in the washing water or the carbonate bath diminish the effect, and .004 per cent. of potassium bromide or .2 per cent. of sodium chloride is sufficient to prevent color sensitizing entirely. No noticeable restraining action occurs, however, if these are present only in the bisulphite.

Numerous other experiments were tried with more or less inconclusive results and the investigation so far has given so many inconsistencies that the writers prefer to pursue the work further before discussing the matter in fuller detail.

ROCHESTER, N. Y.,
May 14, 1920.

---

**On the Application of Interference Methods to Astronomical Measurements.** A. A. MICHELSON. (*Comptes Rendus*, July 5, 1920.)—Attention is directed to an article published in 1896 where the author proposed a method of measuring such astronomical magnitudes as the diameters of planetoids and satellites, the angular distance between double stars and, perhaps, even the diameters of fixed stars. To do this it was suggested that a series of interference bands be formed at the focus of a telescope by the light from two slits, illuminated by the source of light under investigation. These slits were to be located at the ends of a diameter of a large telescopic objective. The fringes would consist of equidistant bands whose sharpness would be greatest for a point source of light. The angular diameter of the source can be obtained from a study of the distribution of the light in the bands. From a knowledge of the manner of distribution of light in the source the arrangement of light in the bands may be calculated and, conversely, from the latter the former may be inferred.

This method was applied to Capella at the Mount Wilson Observatory and gave satisfactory results. The parallax of this star was found to be a little less than .05 sec. of arc. With a base 250 cm. long an accuracy of .001 sec. is to be counted on. In this experiment the slits were actually put near the eye-piece and not in front of the objective. The fringes remained very fixed and clear in spite of much boiling.

Professor Michelson expects this work to continue with an interferometer base about 6 m. long, and, if these preliminary experiments warrant it, with even a longer base.

G. F. S.

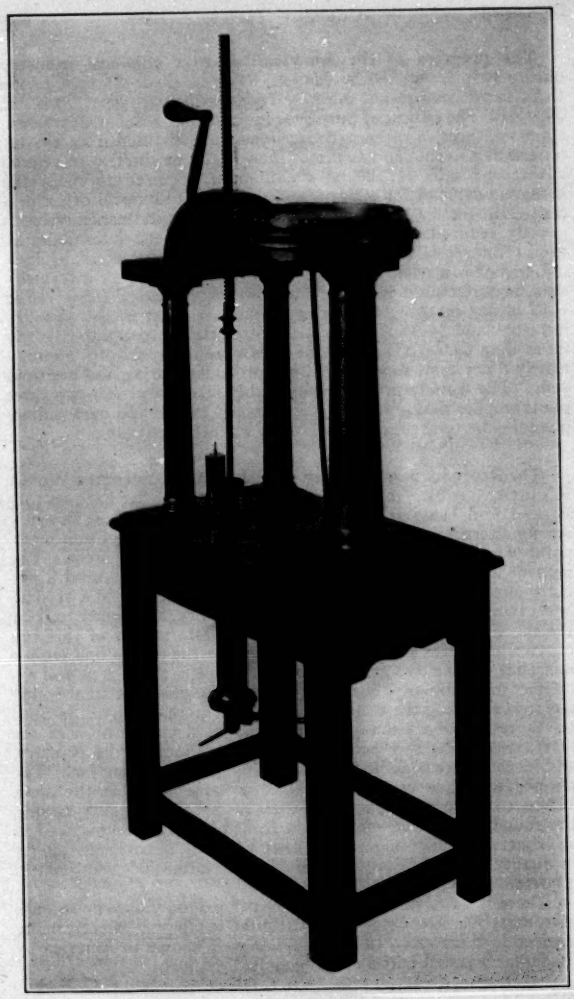
**The progress of the American coal-tar chemical industry** during 1919 is outlined by GRINNELL JONES, Chief Chemist of the U. S. Tariff Commission, in *Journ. Indus. and Eng. Chem.*, 1920, xii, 959–961. The output of pure benzene and pure toluene decreased, since mixtures of these hydrocarbons were distilled from the tar for use as solvents and as motor fuels. The production of refined anthracene falls far below the amount required to meet the domestic demand for alizarin and vat dyes. Removal of anthracene from the tar leaves the pitch so hard that its market value is greatly reduced. The output of certain coal-tar intermediates, used primarily in the manufacture of explosives or of khaki dyes, underwent a marked decrease. The total number of intermediates, manufactured in the United States, increased from 140 in 1918 to 225 in 1919; these compounds are used in the dyestuff industry and in the preparation of synthetic drugs. Slightly more than 63,000,000 pounds of dyes were made in 1919, approximately 8 per cent. more than were produced during the previous year. The dyes improved considerably in quality. Of the three photographic developers—hydroquinone, metol, and para-aminophenol—the total production in 1919 was about 195 tons.

J. S. H.

**The Reproduction of Speech by Galena and Undamped Waves.** P. COLLET. (*Comptes Rendus*, June 7, 1920.)—As far back as 1913 Brazier and Dongier showed that the passage of a sufficiently strong alternating current across the surface of contact of a metal point with a crystal of galena caused a sonorous effect. The present investigator in continuing these experiments used a primary circuit in which undamped waves were set up and a secondary circuit, entirely separate from the primary, and tuned to it by means of a variable condenser. From the terminals of this run off two line wires several meters long. At the end of one wire is a crystal of sensitive galena, while a platinum point is at the extremity of the other. The point touches the galena, thus forming a circuit shunting the condenser. Furthermore, the point is fixed to the centre of a phonograph disc mounted on the horn of the instrument. Let the secondary circuit be tuned to the primary, and let the latter be periodically interrupted by a tuning fork. The phonograph then emits a note of the same pitch as the fork. When a carbon microphone was joined in the secondary circuit, the sounds of a voice speaking in front of it were reproduced in an adjacent room with great intensity and with remarkable fidelity in quality. The singing voice is likewise admirably rendered in all detail.

There is no advantage gained by using loud voices or intense currents. The phenomenon is attributed to thermal effects, being kindred to the Trevelyan rocker made well known in America by Professor Tyndall half a century ago.

G. F. S.



The Priestley air pump

## THE PRIESTLEY AIR PUMP.

PRESENTED BY

**COLEMAN SELLERS, JR., and HORACE W. SELLERS.**

In presenting to the Institute, at its Stated Meeting held Wednesday evening, October 20th, the air pump, once the property of Dr. Joseph Priestley, shown on the opposite page, Mr. Coleman Sellers, Jr., made the following remarks:

*Mr. President, members of The Franklin Institute, ladies and gentlemen:* Before considering this ancient piece of philosophical apparatus, it may add to its interest, if we recall to memory some facts about the remarkable man to whom it formerly belonged.

Dr. Joseph Priestley is recollected chiefly as the discoverer of oxygen; but this accomplishment by no means comprises all of his contributions to science, for he also discovered and experimented with many other gases; but the curious part of the matter is that his scientific work was his avocation, or pastime. By profession, he was a controversial non-conformist clergyman, and a prolific writer and lecturer on theological and other topics. He was an ardent political reformer, and it was to his political activity in an unpopular cause that we owe his exile to America, which has finally brought this historic air pump to a permanent home in the collection of this Society.

Doctor Priestley was born in Yorkshire, near Leeds, March 13, 1733, his father being Jonas Priestley, weaver and dresser of cloth. From 1742 to 1764 he made his home with an aunt who sent him to school, where he learned Latin and Greek. He studied Hebrew in his holidays, so that when he was about sixteen years old, he claimed to have a pretty good knowledge of the ancient languages. He came of a strongly religious family, and it was his intention to enter the ministry; but lung trouble for a time deterred him from this course and he took up the study of modern languages, learning French, Italian and German.

He also studied mathematics, logic and natural philosophy. He learned Hebrew, Chaldee, Syriac, and began to read Arabic. In 1755 he became assistant to a superannuated minister for a compensation of 30 £ sterling per year; and in 1758 he assumed the pastorate of a church near old Chester. His congregation was

small and his duties not burdensome, so he opened a school for about thirty boys and six girls and taught daily from 7 A.M. to 4 P.M. with one hour for dinner and no holidays. After school hours, he had private pupils.

He was now able to buy a few books and even an air pump and an electric machine, and it was through this apparatus that his interest in experimental science began. Incidentally he wrote and lectured on many subjects covering various phases of political economy, government, civil law, logic, anatomy and Hebrew.

During a visit to London, he met and formed a strong friendship with Benjamin Franklin, who, undoubtedly exercised a powerful influence in shaping Priestley's future life. He offered to write a history of electricity, if Doctor Franklin would furnish him the necessary books. This he gladly did, and the work was published in 1767, under the title of "The History and Present State of Electricity, with Original Experiments Illustrated with Copper-plates." Much of this work was original matter, the result of his own experiments, and although the laboratory work and composition were done in the leisure time of a busy clergyman, he was able within a year to send to Doctor Franklin a printed copy of the book. It is worthy of mention that during this year, he spent five hours a day lecturing and took a two months' vacation. This book resulted in his election as a Fellow of the Royal Society.

He was ordained a minister in 1762 and in 1767 became pastor of a congregation in Leeds, at a salary of 100 guineas and a house. His wavering heterodoxy built on a Calvanism, which he did not find convincing, finally settled into Unitarianism and he became one of the chief upholders of heterodoxy in religion and of advanced ideas in politics.

It was at Leeds that he began his work on gases. He started with a study of "fixed air," or carbon-dioxide, and invented soda water, which he suggested might be useful in medicine. He made his first publication on gases in 1772, for which he was awarded the "Copley Medal," and in succession he discovered nitrous oxide, hydrochloric acid gas, nitric oxide, oxygen (in 1774), ammonia gas, sulphur dioxide, silicon fluoride and nitro-sulphuric acid.

It is not to be understood that he recognized these substances by the names we now call them. Oxygen, he knew as "dephlogisticated air." In fact, his philosophy was usually vague and his

reasoning defective, largely perhaps because he was influenced by the prevalent theories of the time, and also it has been suggested, by defects of memory. He often forgot his own notes, and his own publications. In his cheerful way he suggested that this defect was in a measure compensated for by facility in "invention or new and original combinations of ideas." He was much more interested in trying experiments than he was in philosophic deductions from the facts observed.

Priestley was a real discoverer, a brilliant experimenter, an indefatigable investigator. He proved by experiment "that plants are perfectly capable of restoring air injured by respiration"; and he made some measurements of the oxygen absorbed and the carbon-dioxide evolved during respiration.

His busy and useful life was disturbed in 1791 by the Birmingham Riots, which were precipitated by a demand for the repeal of the Test Act. Priestley was active in this movement and also sympathized with the French Revolution. When an attempt was made to celebrate the fall of the Bastile on July 14, 1791, by a public banquet, the hotel was attacked by a mob, which broke the windows and then proceeded to Priestley's church, the "New Meeting," which was sacked and burned, as was also the "Old Meeting." Burning of non-conformists' churches and ministers' houses continued, and Priestley was hurried to London, where he was taken charge of by his friends.

He was offered a pulpit, but felt that his usefulness in England was ended, and he concluded to join his sons in America. He sailed for New York, April 8, 1794, proceeded to Philadelphia and then to Northumberland, Pa. Soon after his arrival in Philadelphia, he was offered the professorship in chemistry at the University, which he declined, and in 1803, it was proposed to make him Provost of the University. This he also declined on account of his advanced age. In America he resumed his scientific work; but made no new discovery of importance, although he published a number of papers. He died February 6, 1804.

When he came to America, he brought with him such of his apparatus as he could, and among other things was the air pump which we have before us.

As to the history of the pump since Doctor Priestley's death: My father's correspondence[1] shows that it was procured by his

[1] Coleman Sellers, E.D., D.Sc. (1827–1907), President of The Franklin Institute, 1870–1874.

brother Charles,[2] who was a friend of the Priestleys, for use in some lectures which he was delivering in Cincinnati. When he left that city about 1849, he turned his philosophical apparatus over to his younger brother Escol,[3] who subsequently removed to southern Illinois.

In 1897, at my father's request, Escol Sellers sent him the air pump, which apparently had never been used while in his possession. Escol wrote that when his brother Charles got the pump, the valves and pump chamber were in bad shape, and he worked a long time to put them in order. He said that on the frame, there was a portion of a bone or ivory name plate, on which they made out the words "Adams" and "London," which they inferred to have been the name of the maker and his address.

We all value this relic highly and my brother[4] and I have felt anxious to transfer our responsibility as its custodians to that association which would most appreciate it, and would give it the best care.

We have concluded that the most appropriate custodian is The Franklin Institute of the State of Pennsylvania, and we feel sure that we are putting the old pump in safe and appreciative hands.

I take pleasure, Mr. President, in delivering to the Institute, over which you so ably preside, this ancient relic and memento of a remarkable man, with the full assurance that it will be guarded and preserved with the respect it deserves.

---

**The Dye-works Chemist.**—According to B. LEECH (*Jour. Soc. Chem. Ind.*, 1920, xxxix, Review, 300–302), the chemist in a British dye-works should have a broad scientific training in a university, supplemented by special training in a technical school. Among the problems which he may have to solve are: plan of buildings and proper materials for their construction; ventilation of the plant; weaving goods "in the gray" and dyeing the woven piece; special treatments, such as weighting of silk and mercerization of cotton; production of artificial fibres; treatment of fabrics to render them fireproof, waterproof, or non-permeable to gases, and many procedures which involve organic colloidal, and biological chemistry, mycology, physics, and engineering.

J. S. H.

---

[2] Charles Sellers (1806–1898).

[3] George Escol Sellers (1808–1899).

[4] Horace Wells Sellers, F.A.I.A.

## NOTES FROM THE U. S. BUREAU OF STANDARDS.*

### STEEL RAILS FROM SINK-HEAD AND ORDINARY RAIL INGOTS.[1]

By George K. Burgess.

[ABSTRACT.]

THE object of this investigation was to determine the relation of ingot practice to the properties of rails from such ingots, and in particular to determine the amount of total discard necessary to obtain rails free from piping and segregation above 12 per cent., which has been rolled from steel made in accordance with varying melting, casting and ingot practices.

To that end 35 ingots, made by the converter process at Hadfield's, Sheffield, England, and cast by the sink-head process with large end uppermost, were shipped to Sparrow's Point, Md., and rolled into rails; these were compared with 15 rail ingots made in the ordinary manner with the small end uppermost. Each sink-head ingot, of about 5300 lb. weight, and deoxidized with aluminum in the mold, represented a separate heat of converter steel, and all the heats and ingots were made in the same manner. The composition and properties of these ingots were of remarkable uniformity. The comparison ingots, of 7300 lb. each, were from three separate open-hearth heats, an additional variation being made in the casting and open-hearth practice for each. Five ingots were selected from each of these three heats. Thus, in reality, comparison was made of four different kinds of steel of very nearly the same composition and physical properties and of two types of ingot form.

The comparison was made by rolling most of the ingots into rails and taking test specimens at each rail cut, as well as from a considerable portion of the upper part, in five-foot steps, of the rail bar from each ingot. In this way there was obtained a detailed physical, chemical and metallographic survey of each ingot, and it was possible to exactly delimit the regions of sound and homo-

* Communicated by the Director.

[1] Technologic Paper No. 178.

geneous, from those of unsound and segregated, steel. Two complete sink-head ingots were cut longitudinally and examined, as also were representative blooms from both sink-head and ordinary ingots.

The results obtained indicate a decided superiority of the sink-head ingots over the comparison ingots as made of three grades of steel (Tables 17 and 18), although the sink-head ingots suffered from the disadvantage of having gone cold before rolling. The Hadfield type of ingot required a total discard of only 18.4 per cent. on the average (13 per cent. top discard to eliminate piping and segregation above 12 per cent.), while the average ingot of the ordinary type for rails required a total discard of 43.9 per cent. (26 per cent. top discard), with great variations dependent upon the furnace and ingot practices.

The comparison ingots from heat M1–M5, made of non-deoxidized rising steel chilled on top of ingot by cast-iron caps, required excessive discard to eliminate positive segregation at the top and negative segregation at the bottom of the ingot, the latter often accompanied by dangerous enclosed pipes.

The second heat (M6–M10), made of rising steel deoxidized with aluminum in the molds, the ingot tops of which were cooled with water, required the least total discard of the three heats. It was more subject to piping and less to segregation than the first heat of ingots made in the usual manner.

The third heat (M11–M15), made of quiet or "killed" steel, was not chilled on top with water nor caps and was deoxidized with aluminum in the molds. The ingots of this heat required an intermediate amount of total discard when compared to the first and second heats; this heat was the only one for which a greater top discard was required to eliminate piping than to eliminate segregation above 12 per cent. One of the ingots of this third heat contained a small pipe at the bottom and all the rails from the middle and bottom of the ingots showed high negative segregation.

The distribution of physical properties throughout the length of each ingot is characteristic not only of the type of the ingot, as sink-head or ordinary, but also of the state of the steel when cast, and of the ingot practice.

It has been established in the foregoing, that after removal of the top discard of 13 per cent., the Hadfield type of sink-head

ingot is free from piping and undue segregation. The ordinary type of ingot, cast small end up without sink-head as is usual for rail ingots, requires an average top discard of 26 per cent. and the remainder of the ingot is liable to contain enclosed piping and excessive segregation. Defective rails, from the middle and bottom portion of the ingot, are not certainly detected by means of existing rail specifications, and as a result of this uncertainty, rails containing pipes or excessive segregation may get into service with disastrous results.

The surface condition of the rails from the sink-head ingots was not so good as for the ordinary ingots, but this is not considered an essential characteristic of rails from such ingots.

The markedly differing characteristics of the three heats of comparison ingots leads one to raise the question whether or no it might be advisable to specify, at least in some degree, the methods of steel manufacture or of ingot practice for rails and similar products on which the safety of the travelling public depends.

While it is not claimed that the use of the sink-head process for the manufacture of ingots will solve all rail problems, it is maintained that its adoption would be a step in the right direction in view of the present heavy casualties and property losses on American Railroads. The necessary changes in mill operations, it is believed, could be made without too great difficulties.

## SULPHUR IN PETROLEUM OILS.[3]

By C. E. Waters.

[ABSTRACT.]

Short accounts are given of the theories concerning the origin of the sulphur and sulphur compounds which are found in crude petroleum. The forms of combination in which the element occurs, their identification and significance are briefly discussed.

Tests for the detection of sulphur are described, and the copper test is shown to be one of great delicacy. Although it will show the presence of very minute amounts of free sulphur or of hydrogen sulphide, it may be of no value at all when the sulphur is in stable organic compounds.

Various methods that have been used for the determination of

[3] Technologic Paper No. 177.

sulphur in oils, and finally a new procedure, are described. The new method is based on the preliminary treatment of the oil with nitric acid saturated with bromine, followed by fusion with a mixture of sodium nitrate and carbonate. Certain sources of error which are inherent in the method and the way in which they can be corrected for, are taken up.

Data obtained by the analysis of certain oils by this and other methods are given. From these it appears that there is no loss of sulphur when oil is treated with nitric acid and bromine, and then ignited with the fusion mixture. The method is recommended for laboratories which do not have a bomb calorimeter.

## INFRA-RED TRANSMISSION AND REFRACTION DATA ON STANDARD LENS AND PRISM MATERIAL.[3]

By W. W. Coblentz.

[ABSTRACT.]

TRANSMISSION curves of quartz, fluorite, rock salt, and sylvite, etc., are given, and the suitability of these substances for prisms, with especial reference to infra-red spectro-radiometry, is discussed. Tabulated data are given of the refractive indices of these materials. The paper discusses also the construction and methods of calibration of spectroradiometers.

## SLUSHING OILS.[4]

By P. H. Walker and Lawrence L. Steele.

[ABSTRACT.]

SLUSHING oils are materials used for protecting bright metal where it is not practical to use paint, varnish, or other fixed coatings. An ideal slushing oil is one which can be easily applied to all kinds of metal surfaces by a variety of methods. It should coat the surfaces with a sufficiently thick and impervious film to exclude moisture and air (to prevent rusting), should remain in position for an indefinite length of time, and yet be completely removable from the surface without undue labor. The material should itself have no corrosive action on any kind of metal.

This paper contains a discussion of properties and methods of testing, most of which were developed in the course of this investi-

[3] Scientific Paper No. 401.

[4] Technologic Paper No. 176.

gation, and summarized results of tests of a number of samples. From a study of numerous laboratory and exposure tests proposed specifications are given. The specifications suggested are based upon properties of the finished product rather than chemical composition. Formulas are given of some satisfactory mixtures, but it is not claimed that these are the best slushing oils that can be made. They are merely cited as examples of easily made preparations which were found to protect metal.

## TESTS OF BOND RESISTANCE BETWEEN CONCRETE AND STEEL.[8]

By W. A. Slater, F. E. Richart and G. G. Scofield.

[ABSTRACT.]

This paper includes reports of three separate series of tests:

1. Tests to determine the effect of preservative coatings on bond resistance of reinforcing bars embedded in concrete.

2. Tests to determine the length of the lap which is necessary in order to secure the effective splicing of bars by lapping in regions of high tension.

3. Tests to determine as to the effectiveness of the anchorage of stirrups in the flanges of T-beams when the loads on the beams are placed in such a way as to put tension across the plane of connection between the flange and the stem of the beam.

The tests with preservative coatings indicate:

1. The maximum bond stress developed by bars which were painted, was generally considerably less than the bond resistance of unpainted bars, but the reduction in maximum bond resistance due to galvanizing and some similar processes was less than that due to painting.

2. Coated deformed bars apparently slipped considerably before the corrugations or lugs reached a firm bearing. After a bearing had been secured the increase of resistance with increased slip was similar to that which took place with uncoated deformed bars.

3. The bond resistance at a slip of 0.001 in. was a much smaller proportion of the maximum bond resistance for coated bars than for uncoated bars.

The tests of beams with lapped bars indicated that:

1. Proceeding along a lapped bar towards its unanchored end

[8] Technologic Paper No. 173.

from the point where the stress in the bar began to diminish the stress lost from this bar was picked up by the other bars of the beam. In a majority of cases a slightly larger amount of the stress lost by the lapped bar was picked up by bars immediately adjacent than by other bars of the beam.

2. A lap of about 48 diameters of the bar was necessary to secure a satisfactory splice. The bars used were ½ in. in diameter and the concrete had a strength of about 5000 lb. per sq. in. Alongside the lapped bars were unspliced bars which extended throughout the total length of the beam.

The pull-out tests of stirrups indicated that right-angle hooks formed by bending through 90° were not as effective for anchorage as right-angle hooks bent through 270°. The anchorage was less effective when the loop formed by the 270° turn was kept empty than when it was filled properly with concrete.

## THE IONIZATION AND RESONANCE POTENTIALS OF SOME NON-METALLIC ELEMENTS.[4]

By F. L. Mohler and Paul D. Foote.

[ABSTRACT.]

MEASUREMENTS of the critical potentials were made in four electrode vacuum tubes by methods similar to those previously used by the authors. The essential features of the methods are the detection of the resonance potential by the occurrence of inelastic electron collisions and the ionization potential by the presence of positive ions. The following table gives the results obtained. The theoretical values for hydrogen are derived from Bohr's theory for atomic and molecular hydrogen.

*Summary of Critical Potentials of Gases and Vapors.*

| Element | Resonance | | Wave-length | Ionization | | Wave-length |
|---|---|---|---|---|---|---|
| | Observed | Theoretical | | Observed | Theoretical | |
| | | | λ in Å | | | λ in Å |
| Phosphorus | 5.80 | ..... | *2130* | 13.3 | ..... | *928* |
| Iodine | 2.34 | ..... | *5300* | 10.1 | ..... | *1220* |
| Sulphur | 4.78 | ..... | *2580* | 12.2 | ..... | *1010* |
| Nitrogen | 8.18 | 8.270 | 1492.8<br>1494.8 | 16.9 | ..... | *730* |
| Oxygen | 7.91 | ..... | *1560* | 15.5 | ..... | *796* |
| Hydrogen | 10.40 | 10.16 | 1215.6 | 13.3 | 13.54 | 911.78 |
| | 12.22 | | | 16.5 | 16.26 | |

Italicized wave-lengths are computed from observed potentials. The others are the spectroscopic values from which theoretical potentials have been computed.

[4] Scientific Paper No. 400.

## POSITIVE AND NEGATIVE PHOTO-ELECTRICAL PROPERTIES OF MOLYBDENITE AND SEVERAL OTHER SUBSTANCES.[1]

By W. W. Coblentz.

[ABSTRACT.]

THE paper gives a description of new observations on positive and negative spectrophotoelectrical reactions in molybdenite, silver sulphide, etc., as dependent upon temperature as well as magnitude and direction of the current through the crystal. It is shown that for radiation of wave-lengths less than 0.7$\mu$ the photoelectrical reaction, in a certain sample of molybdenite, is positive or negative depending upon the magnitude and direction of the current through the crystal. In three appendices data are given: (1) on a frequency relation in the sensitivity bands of molybdenite, (2) a survey of the general spectrophotoelectrical properties of substances and, (3) on thermal radiophonic signalling apparatus.

---

## METALLOGRAPHIC ETCHING: I: FOR COPPER.[2]

By Henry S. Rawdon and Marjorie G. Lorentz.

[ABSTRACT.]

THE investigation of metallographic etching reagents in progress at the Bureau of Standards is illustrated by a detailed account of the etching of the various forms of copper. This metal was chosen as the first type for consideration on account of its industrial importance as well as because its behavior is typical of many of the important industrial alloys of copper, *i.e.*, the brasses and bronzes.

Two general types of etching as defined by the characteristic appearance of the etched surfaces are described. These are designated as "plain" and "contrast." While many reagents as a rule give but one type of etched surface, it is possible to produce either type, as desired, from some reagents. In general, "contrast" etching is most suitable for microexamination at relatively low magnifications. For study of those features of structure for which high magnifications must be used, the plain uncontrasted etched surface is desired.

Oxidation is of fundamental importance in the solution of

[1] Scientific Paper No. 398.

[2] Scientific Paper No. 399.

copper and consequently in the etching of the metal. Many solutions which have a very slight etching action upon copper may be made to etch readily by passing oxygen through them, but solutions which have no solvent action upon copper cannot be made to etch the metal by the addition of oxygen. Ammonium hydroxide, in the absence of oxygen, has an almost negligible action upon copper. Most of the etching solutions in general use consist of an oxidizer in an acid or an ammoniacal solution. A few very powerful oxidizing agents may be used in solution without the addition of either ammonium hydroxide or acid.

In general, aqueous solutions are best for the etching of copper. In a very limited number of cases, however, alcoholic solutions appear to give better results. Typical etching reagents are described, and micrographs are given to illustrate the characteristic features of each.

---

## THE EFFECTS OF CAL AS AN ACCELERATOR OF THE HARDENING OF PORTLAND CEMENT MIXTURES.[1]

By Roy N. Young.

[ABSTRACT.]

THIS paper presents the important details and results of an investigation conducted at the U. S. Bureau of Standards for the purpose of determining the relative value of Cal as an accelerator of the hardening of Portland cement mixtures. While a greater part of the paper deals with the effect of Cal on the early strengths of mortars and concretes, results of miscellaneous tests bearing upon the nature of the material and its effects on setting-time, soundness, etc., are included.

Cal is essentially an oxychloride of lime in a powdered form. It is readily decomposed by water into calcium chloride and calcium hydroxide, the property of accelerating the hardening of Portland cement being due to the former product. The chief advantage of using Cal rather than calcium chloride is the much greater convenience with which it may be handled.

The setting-time of Portland cement mixtures was hastened by the addition of Cal to an extent which is considered very desirable in certain types of concrete construction, especially floor toppings, for the reason that the finishing operation may proceed

[1] Technologic Paper No. 174.

with much less delay. Tests on quick-setting cements indicate that such cements may be made normal by the addition of Cal. It is also shown that unsoundness may be greatly improved or entirely overcome by this treatment.

The early hardening of Portland cement mixtures is hastened by the addition of Cal. The strength of mortars was increased from 40 per cent. to 140 per cent., and the strength of concrete from 52 per cent. to 135 per cent. at the age of two days by an addition of Cal equal to 5 per cent. of the cement. Comparison tests, in which commercial calcium chloride was used as the accelerator, indicate that the general effect of Cal is practically the same as might be expected from the use of equivalent amounts of calcium chloride and hydrated lime.

No conclusions were drawn from very limited corrosion tests, but it is advised that caution be exercised in the use of Cal in concrete containing steel reinforcing if the concrete is to be freely exposed to the weather or excessive dampness.

---

## POURING AND PRESSURE TESTS OF CONCRETE.[10]

**By W. A. Slater and A. T. Goldbeck.**

[ABSTRACT.]

THE tests reported in this paper were carried out for the purpose of obtaining information for use in the construction of concrete ships.

In the pouring test the aim was to determine whether, with the thin shell of the concrete ship, it was possible to pour, properly, concrete of the consistency which was to be used in the construction work, constricted as was the space in the forms by the presence of large quantities of reinforcement. A slab 6 ft. square and 4 in. thick was constructed by standing the form up on one edge and pouring the concrete into the form through windows in the side at three different elevations. By means of grooves in the slabs which were provided in the making of the slab, the slab was broken into heavily reinforced beams each a foot wide and six feet long. These beams were tested to determine something of the quality of the concrete.

This test indicated that the pouring could be carried out suc-

[10] Technologic Paper No. 175.

cessfully under the conditions of ship construction, and it showed that the concrete so obtained was of a high grade.

In the pressure test the pressure of the concrete against the forms was measured. The form used was about 17 feet high. The concrete was introduced through windows placed at intervals in the height of the form. The pressure measuring instruments were placed at various heights, and the readings of pressure taken at various intervals of time after the concrete had been placed.

The maximum pressure against the forms during the pouring of concrete was found to be equivalent to that of a liquid weighing 124 lb. per cu. ft.

The maximum pressure was found to be that due to the head of concrete existing at the end of about 40 minutes after the placing of the concrete. After that time the pressure decreased gradually in spite of an increasing head.

---

**The Radium Content of the Rocks of the Loetschberg Tunnel.** J. H. J. POOLE. (*Phil. Mag.*, October, 1920.)—This tunnel, 14,536 m. in length, runs from Kandersteg to Goppenstein in the Bernese Oberland. Samples of the pulverized rock were heated to about 1100° C. with an equal mixture of sodium and potassium carbonate. The carbon dioxide evolved was absorbed in a soda-lime tube and the residual gases stored. When the decomposition of the rock was completed they were swept back into an exhausted electroscope, from the change of deflection of which the mass of radium present in the gas was inferred. The results are stated by giving the billionths of a gram of radium present in one gram of rock. This radium content is least, .8, for a specimen of quartz schist and greatest, 6.5, for talc felspar schist. When the determinations are averaged for all the specimens of the same kind of rock quartz sandstone is seen to lead the list with a content of 4.3, while anhydrite is lowest with 1.1. The northern end of the tunnel was the poorer in radium. The general mean for all the rocks examined was 2.2.

In the St. Gothard tunnel a very large temperature gradient at one end was coincident with a high radium content of the rocks traversed. In the Loetschberg tunnel neither the thermal gradient nor the radium content varied greatly along the portion examined.

G. F. S.

## NOTES FROM THE RESEARCH LABORATORY EASTMAN KODAK COMPANY.*

### IMAGE CONTRACTION AND DISTORTION ON PHOTOGRAPHIC PLATES.[1]

By F. E. Ross.

THIS is a comprehensive résumé of what has been done previously and of recent investigations. The author deals mainly with the gelatine strains and stresses, and the tendency, when part of a photographic plate has dried and an adjacent region remains wet, for the wet gelatine to be drawn toward the dry portion, and the influence of this tendency upon the resultant relative positions of adjacent images or parts of images, especially when there is any comparatively large mass dry before the adjacent portions, such as the image of the inner portions of the solar corona during a total solar eclipse. The bearing of these distortion effects upon photographic tests of the "Einstein Theory" made during total solar eclipses is taken up; also, the effect upon photographs of double stars and star clusters. It may also influence measurements of the distance apart from adjacent lines in photographs of spectra.

### A CHART METHOD OF TESTING PHOTOGRAPHIC LENSES.[2]

By L. E. Jewell.

[ABSTRACT.]

THE method discussed consists in photographing with the lens under test a chart composed of a large number of similar units, each unit being a radially sectored circular chart, the sectors being alternately black and white, and approximately 5° in angular diameter. The interpretation of the photographs of this chart in terms of the lens aberration is discussed.

* Communicated by the Director.

[1] Communication No. 94 from the Research Laboratory of the Eastman Kodak Company, and published in *Astrophys. J.*, September, 1920, p. 98.

[2] Communication No. 77 from the Research Laboratory of the Eastman Kodak Company, and published in *J. Opt. Soc. Amer.*, 1919, p. 51.

## PHOTOGRAPHIC PHOTOMETRY AND THE PURKINJE EFFECT.[3]

By F. E. Ross.

[ABSTRACT.]

THE increase in diameter of a star's image is proportional to the logarithmic increase of exposure time, and it is proposed that when this is measured with an artificial star the coefficient of increase shall be called the *astrogamma* because of its analogy with the factor which is termed *gamma* in photographic work. Both gamma and astrogamma are dependent upon the wave-length, and photometric measurements of stars made photographically must take into consideration this variation of gamma and astrogamma with wave-length. These effects are considered in the paper. Some results obtained with long exposure where the light source is very weak are also given. The theoretical considerations involved are dealt with here.

---

## ON THE RELATION BETWEEN PHOTOGRAPHIC DENSITY, LIGHT INTENSITY AND EXPOSURE TIME.[4]

By F. E. Ross.

THE equations connecting photographic density with exposure which have been proposed by Abney, Hurter and Driffield, Elder and Channon are critically discussed. In particular certain theoretical objections to the Hurter and Driffield formula are advanced. The author proposes a new formula, the underlying idea being the separation of the grains of an emulsion into different classes, according to their sensitiveness. The thickness of the emulsion is taken into consideration, and is measured not in microns but by number of layers of grains. The mass-action equation is assumed to hold for each group. All formulæ are compared with two types of measured characteristic curves, in one of which the "toe" is prominent, in the other, it is almost absent. It is difficult to include the characteristics of "toe" and "shoulder" in any simple algebraic formulæ. These peculiarities point to the existence of secondary phenomena.

---

[3] Communication No. 95 from the Research Laboratory of the Eastman Kodak Company, and published in *Astrophys. J.*, Sept., 1920, p. 86.

[4] Communication No. 93 from the Research Laboratory of the Eastman Kodak Company, and published in *J. Opt. Soc. Amer.*, Sept., 1920, p. 255.

The paper concludes with a discussion of the reciprocity law, the necessity of further experimental work being emphasized. The laws of Schwarzschild and Kron are discussed and compared with the reciprocity law. Assuming Schwarzschild's law correct, two simple methods of obtaining the parameter P in the formula are given.

---

**Home Production of Rare Carbohydrates.** (*A. C. S. News Service, Bul. 275.*)—A number of carbohydrates of special nature are required in modern bacteriologic work, and up to comparatively recent times the supply of pure forms was entirely from Germany. American manufacturers have been giving attention to the matter, and it now appears that the domestic supply is ample and satisfactory. These products have to be of high purity in order to furnish the closely standardized media now employed in bacteriologic and pathologic research, and some of the procedures are elaborate and difficult, so that the products are costly. Fortunately, the amounts used in ordinary work are quite small, so that the operating cost in their use is not great.

Among the carbohydrates now available from American houses are: dulcitol, mannose, xylose, inulin, levulose and raffinose. Dulcitol is the most expensive, being quoted at about one dollar per gram. In the bulletin from which the above data are taken there is some persiflage about the source of mannose being the manna alluded to in the Bible, but of that statement the less said the better.

H. L.

**Commercial Air-Transport.**—The flying machine is now so familiar an object that it attracts but little attention as it passes overhead. The possibility of rapid and lofty flight with heavier-than-air machines is no longer questioned by any one. The days are past when such eminent scientists as Lord Kelvin and Simon Newcomb could express the view that such flight by any but a very small machine is impossible. Langley, to whom is due the initiative of practical aviation, worked amidst most discouraging conditions; present-day experts in the field have no need to discuss either with scientists or the public at large the practical problems of flying. Nevertheless, some disappointment is felt by many that the airplane has not been employed more extensively in the arts of peace. During the last few years we have heard much of its power as a destroying agent in war, its work in this field partaking of both dramatic and brutal features, and since the establishment of peace, or, at least, armistice, a good deal has been done in the way of sport and some little in the way of express service, principally in connection with the post office.

H. L.

**Excessive Gasoline Waste.** (*A. C. S. News Service, Bull. 276 C.*)—Dr. A. C. FIELDNER, Supervising Chemist, U. S. Bureau of Mines, Exp. Station at Pittsburgh, Pa., has found as result of extensive investigations that nearly one-third of the gasoline used in automobiles is wasted. The experiments were conducted to ascertain the pollution of the atmosphere of the vehicular tunnels now under construction in New York City, by motor vehicles. The experiments were made under the same conditions as apply to actual traffic. The chief cause of the waste is a too rich gasoline mixture. An improved form of carburetor should be devised, which should be practically automatic in operation. On the basis of 34 cents per gallon, American automobilists are using about $115,000,000 worth of gasoline a year, of which, according to Doctor Fieldner, about $34,000,000 could be saved by improved methods, largely in better construction and operation of carburetors.

H. L.

**A New Determination of Avogadro's Constant.** J. CABANNES. (*Comptes Rendus*, November 2, 1920.)—Monochromatic light from mercury vapor was sent through a mass of argon. 98.5 per cent. of the light diffused laterally by the gas was found to be polarized. On account of this it was permissible to regard the molecules of argon as spheres and to make use of the formula derived by Lord Rayleigh. This gave for the value of Avogadro's constant $(6.90 \pm .25) \times 10^{23}$. Millikan found $(6.07 \times 10^{23})$ and Perrin from his studies of Brownian motions obtained $(6.85 \times 10^{23})$ for this constant. It is interesting that such widely differing methods lead to such concordant results.

G. F. S.

**International Atomic Weights for 1921.**—The Report of the International Committee on Atomic Weights for 1920–1921 (*Jour. Am. Chem. Soc.*, 1920, xlii, 1761–1764) makes but one change in the international table; the atomic weight of scandium is now given as 45.1, instead of the old value 44.1. The atomic weights of the other 82 elements are unchanged.

J. S. H.

**Reaction Between Carbon Bisulphide and Ammonium Carbonate.**—This reaction has been studied by FRANÇOIS A. GILFILLAN of Yale University (*Jour. Am. Chem. Soc.*, 1920, xlii, 2072–2080). At a temperature of 160° C., these two compounds interact, and yield a mixture of ammonium thiocyanate and thiourea; the equilibrium depends upon the concentration of the reagents used, temperature, time of heating, and concentration of the hydrogen sulphide formed in the reaction.

J. S. H.

## NOTES FROM THE U. S. BUREAU OF CHEMISTRY.*

### A CHEMICAL STUDY OF FROZEN FISH IN STORAGE FOR SHORT AND LONG PERIODS.[1]

By E. D. Clark and L. H. Almy.

[ABSTRACT.]

BLUEFISH, *Pomatomus saltatrix,* and weakfish, *Cynoscion regalis,* frozen by commercial methods and stored at 15° F., when "glazed" with an ice covering, remained palatable and wholesome during storage for 16 and 13 months, respectively, but became unpalatable after 2 years. The uneviscerated weakfish were in practically the same good condition after 13 months' storage as were the eviscerated fish. Unglazed weakfish dried out rapidly and were considered unmarketable at the end of 4 months.

The changes in the chemical composition of these fish during storage were slight in most instances. In the bluefish water-soluble, coagulable, and proteose nitrogen decreased, together with the iodin number of the extracted fat, while the nitrogen of amino-acids, ammonia, and amines, and the acid value of the fat increased. In the weakfish, water-soluble and coagulable nitrogen increased at first, then decreased in all fish except the unglazed samples, in which the changes were irregular; proteose nitrogen underwent no distinctive change; amino-acid nitrogen increased slightly; increase of ammonia and amine nitrogen was marked in the glazed uneviscerated fish and but slight in the other lots; the acid value of the extracted fat increased and the iodin number decreased in all samples.

### COMMERCIAL HYDRASTIS (GOLDENSEAL).[2]

By Arno Viehoever.

[ABSTRACT.]

HYDRASTIS is one of the most important domestic drugs, and is also extensively exported. Since it is a very expensive drug, its

* Communicated by the Chief of the Bureau.

[1] Published in *Ind. Eng. Chem.,* 12 (1920), 656.

[2] Published in *J. Am. Pharm. Assoc.,* 9 (1920), 779.

price per pound now being $5.70 to $5.80 for the whole drug, and $6.50 to $6.75 for the powdered drug, it is especially important that it should be in a satisfactory condition.

Thirty-two samples, both whole and ground, of Hydrastis collected in interstate trade, were examined, and, in addition, three of known origin, which had been carefully collected and washed. The results of the examination were as follows:

1. The proportion of rhizome to roots varied considerably in the samples examined.
2. The alkaloid content was found to be higher in the rhizomes than in the roots.
3. The raising of the required minimum alkaloid content to 2.75 per cent. of ether-soluble alkaloids is suggested.
4. Samples collected in interstate trade, while containing sufficient alkaloid, as required by the United States Pharmacopœia, contained rather generally amounts of mineral matter greatly in excess of that naturally present.
5. The total and acid-insoluble ash in the roots was found to be higher than in the rhizomes.
6. A maximum limit of 8 per cent. for total ash, and 3 per cent. for acid-insoluble ash is proposed.

---

**The Spectrum of the Green Ray.** A. Danjon and G. Rougier. (*Comptes Rendus*, October 26, 1920.)—Sometimes a second or two before the sun has completely set the portion of the disk still above the horizon assumes a green color. A similar effect occurs at sunrise. This is known as the "green ray." There have been two theories to account for it. Anomalous dispersion is invoked by one while by the other regular dispersion is regarded as the cause. According to the latter, the atmosphere of the earth forms a vertical spectrum of the sunlight. The blue and violet parts are absorbed in traversing the long stretches of air, and thus the green is left at one end of the spectrum and is the last color seen. One of the authors noticed before the war that the setting sun viewed through a telescope showed, under favorable conditions, a green fringe around its upper half. This was visible for the last ten minutes before sunset. Utilizing this long duration the investigators have photographed the spectrum of the green fringe. It is found to differ from the spectrum of the setting sun only in lacking the red which has been deviated by atmopsheric dispersion. No trace of anomalous dispersion appears. The experiments were conducted on the platform of Strasbourg Cathedral, France.

G. F. .S.

## NOTES FROM THE U. S. BUREAU OF MINES.*

### STRUCTURE IN PALEOZOIC BITUMINOUS COALS.

By Reinhardt Thiessen.

CONCEPTIONS of the origin, composition and general nature of coal differ so widely that to determine the real extent of actual knowledge is no easy matter. The chemist finds difficulty in attacking the chemical problems and the engineer lacks a broad basis for the study of his problems. Thus the economical and efficient utilization of coal in the industries suffers from a lack of knowledge of the composition of coal itself.

Most scientists interested in this matter appear to have had only a vague idea of the components of coal and their origin. During the past century many correct observations have been recorded, but through false or inadequate interpretations many contradictory conclusions have been drawn, resulting in great confusion. Also, the requisite knowledge of chemistry or of other sciences necessary to true interpretations was often lacking. For example, speculation as to the true meaning of the bright and dull bands which are seen in all ordinary bituminous coals, and were observed even by the earliest investigators, has led to the forming of many false theories, such as the one that these bands were due to alternating climatic changes during the period of coal formation. If geologists had known the true nature of coal they would not have made this mistake.

The Bureau of Mines, in the endeavor to clear up some of the confusion that exists and fill existing gaps in our knowledge of the composition and origin of coal, has for some years been conducting at its Pittsburgh station, a microscopic study of coal.

This work has involved the perfecting of a method for preparing the thin sections for direct observation under the microscope or for the preparation of photomicrographs. In these sections the coal is revealed in its original chemical and physical condition and natural color. Some sections have been prepared

* Communicated by the Director.

thin enough to permit photography at a magnification of 2000 diameters. Much skill, experience, and patience are needed, however, to prepare them.

The results of the first series of observations appeared in Bulletin 38 of the bureau, entitled: "The Origin of Coal," by David White and Reinhardt Thiessen, in 1914.

The immediate cause that led the writer of this article to investigate further the composition of coal and the origin and structure of its constituents was that when an attempt was made to discover the cause of the coking or non-coking properties of coal a more exact knowledge of the nature of coal was required. Accordingly, samples were collected from representative bituminous coals in Pennsylvania, Indiana, Illinois, and Alabama, as well as various other localities in this country and in other countries. Results of this later study are presented in Bulletin 117, recently issued by the bureau, entitled: "Structure in Paleozoic Bituminous Coals," by Reinhardt Thiessen.

---

## COAL-WASHING EXPERIMENTS AT SEATTLE STATION.

**By Earl R. McMillan.**

COAL seams in the Northwest are rarely free from "bone" and shale or clay partings, are nearly all folded and faulted, dip at various angles, and in some cases are badly broken. In mining seams the raw coal as it comes to the surface often contains as much as 50 per cent. by weight of waste. The Bureau of Mines experiment station at Seattle is conducting washing tests of this coal in order to improve it for commercial use. Since the process of coal-washing depends upon differences in specific gravity between the clean coal and its associated impurities, the first problem undertaken was that of determining the specific gravity of the clean coal and of each parting of impurity contained in a large number of representative seams.

It was found that the freshly mined coal is practically saturated with water, this being observed in samples varying from low-grade subbituminous up to semianthracite, all of which are found in the State of Washington. The specific gravity values determined for the cleanest particles varied from 1.28 to 1.35; in general, the higher the rank of the coal the higher the specific gravity.

The raw coal contained a relatively small proportion of free particles of clean coal, the greater proportion being bone, shale, and clay particles, with specific gravities ranging from 1.35 to 2.65. The float-and-sink method was used to determine the proportions of clean coal and of the heavier impurities, as well as the relations between their specific gravity and ash content. As in use of this method of testing, difficulty arose in obtaining accurate and consistent results, it was necessary to standardize the method by a long series of careful tests. In the standardization tests it was found that the coal must be kept saturated with water throughout in order to obtain accurate (and comparable) results. In order to test large samples quickly and accurately, a float-and-sink machine was built and, with experience, improved to a very satisfactory degree.

The prime object of the tests is to devise washing methods that will prevent loss of good coal and easily and cheaply remove the dirt. In working out this problem, coöperative studies have been made with several of the coal-mining companies and gratifying results obtained. For example, one company in the State of Washington for years has been discharging sludge into a pile recently estimated at 150,000 tons. As a result of washing tests by the Bureau engineers, the mine owners are erecting a washing plant equipped with five tables for treating the entire pile. The material now has a recoverable value of $2 per ton, or a total of $225,000.

At the plant of another coal company the raw nut coal contained 24 per cent. ash. After washing in a jig, the washed coal was found to contain 23 per cent. ash, and 15 per cent. of the feed was discharged as a washery refuse. After several days experimenting with the jig, using the float-and-sink test as a control, the ash in the washed coal was reduced to 16 per cent., with a recovery of 86 per cent.; 92 per cent. of the washed coal and 25 per cent. of the refuse floated on a 1.55 solution, the loss of good coal in the refuse being reduced from 9.3 per cent. to 3.5 per cent. of the feed. Since approximately 250 tons a day are washed on this jig, the saving amounts to about 17 tons a day, which, at $4 per ton, is worth $68, also the services of two pickers were dispensed with, effecting a further saving of $13.60 a day. The results are given in more detail in a recent report published by the bureau.

## VANADIUM.

**By J. P. Bonardi.**

LEACHING tests of a sample of roscoelite ore, conducted at the Bureau's experiment station at Golden, Colo., has shown that chloridizing roasting, followed by sulphuric acid leaching, using about 250 pounds of acid per ton of ore, and then leaching with water, will recover more than 90 per cent. of the vanadium content. Leaching with water, without the use of acid, yields only 60 to 70 per cent. of the vanadium.

On another ore consisting of vanadinite with a little descloizite, chloridizing and nitre-cake roasts, followed by water and dilute acid leach, extracted more than 90 per cent. of the vanadium. In the chloridizing roast the lead is volatized and can be recovered. Concentration experiments showed that by simple panning of −40 mesh material, with no attempt at classification, a concentrate is produced containing 11.4 per cent. $V_2O_5$ and 43.4 per cent. Pb, with a recovery of more than 75 per cent. of both the vanadium and the lead.

---

**Wind Turbines.** E. WEISS. (*La Nature*, October 23, 1920.)—The deliberate and scientifically executed destruction of the French coal mines by the German invaders followed by their failure to deliver coal to France as they had promised to do in the Treaty of Versailles has naturally constrained French engineers to seek to employ other sources of energy. Plans have been devised for utilizing the energy of the tides, of the waves of the sea, and of the river Rhone. At the last Paris fair a wind turbine was shown. The turbine surmounted a framework about 45 feet high. Its axis was vertical as were the blades which had a length of roughly 18 feet. By the words "Houille Azur" on the structure the title "Blue Coal" is preëmpted for wind power, just as "White Coal" is understood to mean water power. The wind turbine lends itself well to the operation of dynamos with accompanying batteries of accumulators to carry the load while the wind is not blowing. It seems likely that it may be applied also to the operation of irrigation pumps, electric furnaces, small shops and lighthouses. The design must take into account the facts that in the Paris district the longest stretch of wind calm is 60 hours and that in a year there 245 days on which the speed of the wind is 4 metres or more per second.

G. F. S.

# THE FRANKLIN INSTITUTE.

*(Proceedings of the Stated Meeting held Wednesday, November 17, 1920.)*

HALL OF THE FRANKLIN INSTITUTE,
PHILADELPHIA, November 17, 1920.

PRESIDENT DR. WALTON CLARK *in the Chair.*

Additions to membership since last report, 4.

Reports of progress were presented by the Committee on Library and the Committee on Science and the Arts.

The Chairman announced that the first business of the meeting was the presentation of the Elliott Cresson Medal and recognized Mr. Charles Penrose, Chairman of the Committee on Science and the Arts, who introduced Dr. W. L. R. Emmet, to whom had recently been awarded the Elliott Cresson Medal for his contributions to the art of ship propulsion.

Mr. Penrose said:

"*Mr. President.* Due, as you know, to the generosity and constructive forethought of various gentlemen interested in the objects of the Institute, the Institute's Committee on Science and the Arts is enabled to recognize invention and discovery in physical science by means of awards and medals appropriate to the importance of the particular invention or discovery brought to its attention.

"Tonight you are asked to present the Institute's Elliott Cresson Medal to Dr. W. L. R. Emmet in recognition of his notable contributions to the art of ship propulsion.

"Founded some three-quarters of a century ago, this medal has been awarded for such inventions as the telephone, telautograph, typesetting machines, the aeroplane, and multiplex telephony and telegraphy by guided ether circuits.

"With such inventions as these it is felt that Dr. Emmet's electrical method of propelling ships takes rank.

"It would require more time than is allowed me to adequately outline his elegant and epoch-making contributions to an all-important art developed through long and intensive periods of brilliant and successful experimentation. Nor, indeed, is it necessary here as the Institute's report dealing with the subject serves well for purposes of record.

"I quote, however, the report's concluding paragraph:

"'After a careful consideration and study of Dr. Emmet's work relating to ship propulsion, the Institute is of the opinion that it deserves the highest award in its gift for the recognition of inventions of signal importance, and awards to Dr. W. L. R. Emmet the Elliott Cresson Medal in recognition of his "Notable Contributions to The Art of Ship Propulsion."'"

The Chairman presented the Medal and accompanying documents to Dr. Emmet, who expressed his thanks for the honor conferred upon him.

Dr. George A. Hoadley was then recognized and presented Messrs. G. L. Kothny and Robert Suczek, of Philadelphia, who had been awarded the Edward Longstreth Medal of Merit for their Radojet air pump.

Dr. Hoadley said:

"*Mr. President:* Two Philadelphia engineers have invented and put into successful operation a device by means of which expanding steam at a high velocity is used for the removal of air from a zone of high vacuum, a new and extensively used form of jet condenser.

"This device is called the Radojet Air Pump. It consists of two steam ejectors working in series. In the first stage a group of axially directed nozzles is used, while in the second stage steam is delivered radially through an annular nozzle, leaving it in the form of a sheet having a high velocity.

"The apparatus is small in volume, light in weight and has proved itself efficient in service.

"In recognition of the merits of this invention The Franklin Institute has awarded the Edward Longstreth Medal of Merit to Mr. G. L. Kothny and Mr. Robert Suczek, the inventors.

"I have the honor, Mr. President, to present Mr. Kothny and Mr. Suczek."

The Chairman presented the medals and accompanying documents to the inventors. Mr. Kothny expressed his thanks for the honor conferred upon him and his colleague.

Mr. Nathan Hayward offered the following resolution, which was seconded by Dr. Hoadley and passed unanimously:

*Resolved*, That a Committee be appointed to nominate Officers and Managers for the year 1921.

Mr. Charles Day offered the following resolution:

*Resolved*, That a Committee of five members of the Institute be appointed, approved by the Board of Managers, with the President and Secretary of the Institute as ex-officio members, to carry into effect the intent and purposes of the Henry W. Bartol bequest to The Franklin Institute.

This motion was duly seconded by Mr. Charles E. Bonine and unanimously carried.

The paper of the evening was presented by Dr. Charles A. Culver, of the Canadian Independent Telephone Company, Limited, Toronto, Canada, on "Guided Wave Telephony." Consideration was given to the fundamental principle and limitations involved in high frequency multiplex operation and the history of the invention and early development of this new means of communication was briefly outlined. Current practical engineering methods were discussed, including special applications to hydro-electric power line operation and train dispatching. The problem of attenuation and required power was also treated. The subject was illustrated by lantern slides and a working demonstration given of communication by means of guided wave telephony superimposed upon a physical telephone circuit. A vote of thanks was extended to the speaker and the meeting adjourned.

R. B. OWENS,
*Secretary.*

## COMMITTEE ON SCIENCE AND THE ARTS.

*(Abstract of Proceedings of the Stated Meeting held Wednesday, November 3, 1920.)*

HALL OF THE INSTITUTE,
PHILADELPHIA, November 3, 1920.

MR. CHARLES PENROSE *in the Chair.*

The following report was presented for final action:

No. 2753: Sterilization by High Pressure. The Edward Longstreth Medal of Merit to Professor B. H. Hite, of Morgantown, West Virginia.

The following reports were presented for first reading:

No. 2712: "Sponsel" Calibrating and Gun Barrel Straightening Machine. Advisory.

No. 2760: Ankyra Ankor Bolt. Advisory.

R. B. OWENS,
*Secretary.*

---

## SECTIONS.

*Section of Physics and Chemistry.*—A meeting of the Section was held in the Hall of the Institute on Thursday evening, October 28, 1920, at eight o'clock, Dr. George F. Stradling in the chair. The minutes of the previous meeting were read and approved.

Ludwik Silberstein, Ph.D., of the Research Laboratory, of the Eastman Kodak Company, Rochester, New York, delivered an address on "Spectrum Analysis and Its Application to the Study of Atomic Structure," which discussed the modern theories of line spectra from the viewpoints of Plank's quantum concept and Bohr's assumption of stationary orbits.

After a discussion of the paper, a vote of thanks was extended to Dr. Silberstein; and the meeting adjourned.

JOSEPH S. HEPBURN,
*Secretary.*

*Electrical Section.*—A meeting of the Section was held in the Hall of the Institute on Thursday evening, November 4, 1920, Mr. William H. Gartley in the Chair. The paper of the evening, entitled "The Application of Radio to Navigation Problems," was presented by Admiral W. H. G. Bullard, Director, Naval Communication Service, Navy Department, Washington, D. C. Consideration was given to the application of radio telegraphy in the determination of the time and hence the longitude at sea; in furnishing weather reports to vessels at sea and the location and presence of icebergs; in determining the position of ships at sea, and aircraft by means of the radio direction finder; in directing ships through narrow channels by the directive pilot-cable. The application of the radio compass to determine presence of thunderstorms and forecasting their presence and approximate local direction was also described, as well as the determination of position

of ships at sea in thick weather by means of underwater radio signals. Mention was also made of the application of radio to meteorology—for the collection and distribution of synoptic reports throughout the world, of special benefit in forecasting weather for long-distance aircraft flights and consequently safety in sea flying. The subject was illustrated by lantern slides. After a brief discussion a vote of thanks was extended to the speaker and the meeting adjourned.

R. B. OWENS,
*Secretary.*

---

## MEMBERSHIP NOTES.

### ELECTIONS TO MEMBERSHIP.

(*Stated Meeting, Board of Managers, November 10, 1920.*)

RESIDENT.

MR. GEORGE E. EWE, 353 East Walnut Lane, Germantown, Philadelphia, Pennsylvania.

NON-RESIDENT.

DR. CHARLES E. CASPARI, 2108 Locust Street, St. Louis, Missouri.

MR. A. R. STEVENSON, JR., 6 Union Street, Schenectady, New York.

ASSOCIATE.

MR. WALTER C. HESS, 1813 Erie Avenue, Philadelphia, Pennsylvania.

### CHANGES OF ADDRESS.

MR. T. LEES BARTLESON, 305 West 19th Street, Wilmington, Delaware.

MR. J. VAUGHAN BOSTWICK, 426 Montgomery Avenue, Haverford, Pennsylvania.

MR. STERLING H. BUNNELL, 103 Park Avenue, New York City, New York.

MR. R. E. GLOVER, 1022 Jackson Street, Wilmington, Delaware.

MR. F. W. HARTZEL, 932 Cheltenham Road, Philadelphia, Pennsylvania.

MR. F. P. HOWE, 242 South 17th Street, Philadelphia, Pennsylvania.

MR. J. P. JEFFERSON, P. O. Box 645, Santa Barbara, California.

MR. M. C. KENNEDY, Vice-President Pennsylvania Railroad Company, Broad Street Station, Philadelphia, Pennsylvania.

MR. CHARLES W. G. KING, 125 Linden Avenue, Rutledge, Delaware County, Pennsylvania.

MR. GEORGE MACLEAN, 208 West 88th Street, New York City, New York.

MR. B. B. MILNER, in care of Frazer and Company, 30 Church Street, New York City, New York.

MR. S. PEACOCK, Wheeling Steel Building, Wheeling, West Virginia.

COLONEL SAMUEL REBER, 27 Van Dam Street, New York City, New York.

DR. GEORGE D. ROSENGARTEN, 115 Broadway, Point View, Miami, Florida.

MR. FREDERICK W. SALMON, Montour Falls, New York.

MR. C. P. SCHLICKE, 31 Plaza Street, Brooklyn, New York.

MR. H. R. VAN DEVENTER, Westinghouse Electric and Manufacturing Company, East Pittsburgh, Pennsylvania.

MR. W. CHATTIN WETHERILL, Department of Mechanical Engineering, University of Pennsylvania, Philadelphia, Pennsylvania.

## NECROLOGY.

**Mr. Daniel Christy,** 1214 South 5th Street, Philadelphia, Pennsylvania.
**Mr. Charles A. West,** 14 Fulton Street, Boston, Massachusetts.

---

## LIBRARY NOTES.

### PURCHASES.

CLARK, M.—Determination of Hydrogen Ions. 1920.
DEAN, B.—Helmets and Body Armor in Modern Warfare. 1920.
HULTGREN, A.—Metallographic Study of Tungsten Steels. 1920.
LEMPFERT, R. G. K.—Meteorology. No date.
MARSHALL, A.—Dictionary of Explosives. 1920.
SHERBONDY, E. H. and G. D.—Text-Book of Aero Engines. 1920.
VON ROHR, M.—Geometrical Investigation of the Formation of Images in Optical Instruments. 1920.
WINN, W.—Timbers and Their Uses. 1919.
WITHAM, G. S.—Modern Pulp and Paper Making. 1919.

### GIFTS.

Adams Company, Catalogue No. 58. Dubuque, Iowa, 1920. (From the Company.)
Alabama Geological Survey, Bulletin No. 22, Petroleum Possibilities of Alabama, 1920. (From the Survey.)
Alberger Pump and Condenser Company, Bulletin No. 29. New York City, N. Y., 1920. (From the Company.)
Alexander Bros., Eleven Pamphlets. Philadelphia, Pa., 1920. (From the Company.)
American Cast-Iron Pipe Company, Prepared Joint Electric Cast-Iron Pipe. New York City, N. Y., 1920. (From the Company.)
American Institute of Electrical Engineers, Transactions, Vol. xxxviii. New York City, N. Y., 1919. (Given by Dr. R. B. Owens.)
American Locomotive Company, Catalogue No. 10050. New York City, N. Y., 1920. (From the Company.)
American Tool Works Circular No. 4. Cincinnati, Ohio, 1920. (From the Company.)
Anti-Corrosion Engineering Co., Water That Won't Rust. New York City, N. Y., 1920. (From the Company.)
Association Francaise pour l'Avancement des Sciences, Conferences 1918–1920. Paris, France, 1920. (From the Association.)
Automatic Fuel Saving Company, Automatic Combustion Control. Philadelphia, Penna., 1920. (From the Company.)
Baldwin Chain and Manufacturing Company, General Catalogue, Edition F. Worcester, Mass., 1920. (From the Company.)
Barnes, William O., The Barnes Precision Cutter Grinder. Leominster, Mass., 1920. (From Mr. W. O. Barnes.)

Benjamin Electric Manufacturing Company, A Big Thought in a Big Way. Chicago, Illinois, 1920. (From the Company.)

Bernitz Furnace Appliance Company, Clinker-Proof Furnace Walls. Boston, Mass., 1920. (From the Company.)

Black & Decker Manufacturing Company, Electric Air Compressors. Baltimore, Maryland, 1920. (From the Company.)

Blaw-Knox Company, Transmission Towers. Pittsburgh, Penna., 1920. (From the Company.)

Bond, Charles Company, Catalog F-10. Philadelphia, Penna., 1920. (From the Company.)

Brooke Tool Manufacturing Company, Limited, Brooke Tools. Birmingham, England, 1920. (From the Company.)

Brown Instrument Company, Catalogue. Philadelphia, Penna., 1920. (From the Company.)

Bruce Peebles and Company, Current Dynamos and Motors. Pittsburgh, Penna., 1920. (From the Company.)

Busch-Sulzer Bros. Diesel Engine Company, Two-Cycle Type C Diesel Engines. London, England, 1920. (From the Company.)

Butterfield and Company, Division Catalogue 18. Derby Line, Vermont, 1920. (From the Company.)

California State Mining Bureau, Mines and Mineral Resources of Sierra County. San Francisco, 1920. (From the Bureau.)

Carnegie Institute of Technology, General Catalogue, 1920. Pittsburgh, Penna., 1920. (From the Institute.)

Carpenter, George B., & Co., Catalogue No. 110. Chicago, Ill., 1920. (From the Company.)

Chicago Belting Company, Catalog and Belting Reference Book. Chicago, Ill., 1920. (From the Company.)

Chicago Flexible Shaft Company, The Melting Pot. Chicago, Illinois, 1920. (From the Company.)

Chicago, Pneumatic Tool Company, Bulletin 504. New York City, N. Y., 1920. (From the Company.)

Detroit Hexagon Drill Company, Three Pamphlets. Detroit, Michigan, 1920. (From the Company.)

Diamond State Fibre Company, Diamond Fibre and Its Uses. Bridgeport, Penna., 1920. (From the Company.)

Driver-Harris Company, Case Carbonizing. Harrison, N. J., 1920. (From the Company.)

Du Pont Chemical Company, Catalog. Wilmington, Delaware, 1920. (From the Company.)

Duquesne Burner Service Company, Combustion Unit. Pittsburgh, Penna., 1920. (From the Company.)

Dwight Manufacturing Company, Catalogue No. 10. Chicago, Illinois, 1920. (From the Company.)

Economy Electric Devices Company, Bulletin No. 70. Chicago, Ill., 1920. (From the Company.)

Engineer Company, Modern Practice in Combustion Control. New York City, N. Y., 1920. (From the Company.)

Eureka Battery Company, Catalogue No. 10. Chicago, Ill., 1920. (From the Company.)

Fairbanks Company, Catalogue 916. New York City, N. Y., 1920. (From the Company.)

Galvanizing Corporation of America, Electro Galvanizing. Brooklyn, N. Y., 1920. (From the Corporation.)

Golden Anderson Valve Specialty Company, Catalogue No. 20. Pittsburgh, Penna., 1920. (From the Company.)

Greenfield Machine Company, Catalogue No. 7. Greenfield, Mass., 1920. (From the Company.)

Hammond Steel Company, Catalogue No. 20. Syracuse, N. Y., 1920. (From the Company.)

Hays School of Combustion, Opportunities for the Combustion Expert. Chicago. Illinois, 1920. (From the School.)

Holophane Glass Company, Inc., Book No. 340. New York City, N. Y., 1920. (From the Company.)

Huyck, F. C. & Sons, Two Related Industries. New York City, N. Y., 1920. (From F. C. Huyck & Sons.)

Hydro-Electric Power Commission, Twelfth Annual Report for the Year Ending October 31, 1919. Toronto, Canada. (From the Commission.)

Ilg Electric, Ventilating Company, Heating Problems. Chicago, Illinois, 1920. (From the Company.)

Iron and Steel Institute, Journal Vol 101. London, England, 1920. (From the Institute.)

Jackson Compressor Company, Rotary Motion. Denver, Colorado, 1920. (From the Company.)

Jeffrey Manufacturing Company, Bulletin Nos. 288 and 309. Columbia, Ohio, 1920. (From the Company.)

Jellett, Stewart A., Company, The Final Test. Philadelphia, Penna., 1920. (From the Company.)

Jones A. A., & Shipman, Ltd., Better Tools. Toronto, Canada, 1920. (From Jones & Shipman.)

Lehigh University, Catalogue. Bethlehem, Penna., 1920. (From the University.)

Link-Belt Company, Book No. 224, 345 and 352. Chicago, Illinois, 1920. (From the Company.)

Locomotive Superheater Company, Bulletin T-6, Data Book for Engineers. New York City, N. Y., 1920. (From the Company.)

Mason Regulator Company, Catalogue No. 60. Boston, Mass., 1920. (From the Company.)

Metric Metal Works, Bulletin No. 114. Erie, Penna., 1920. (From the Works.)

Michigan Central Railroad Company, Seventy-fourth Annual Report for the Year Ending December, 1919. (From the Company.)

Moltrup Steel Products Company, Catalogue No. 1. Beaver Falls, Penna., 1920. (From the Company.)

National Aniline and Chemical Company, Information Regarding "National Service and Products," New York City, N. Y., 1920. (From the Company.)

National Lamp Works of General Electric Company, Twenty-four Bulletins. Cleveland, Ohio, 1920. (From the Company.)

New York Board of Water Supply, Fourteenth Annual Report. New York City, N. Y., 1920. (From the Company.)

Niles-Bement-Pond Company, Right Line Radial Drill. Philadelphia, Pa., 1920. (From the Company.)

Packard Electric Company, Bulletin Nos. 207–208. Warren, Ohio, 1920. (From the Company.)

Pangborn Corporation, Bulletin No. 50. Hagerstown, Maryland, 1920. (From the Corporation.)

Pennsylvania Railroad Company, Seventy-third Annual Report, 1919. Philadelphia, Penna., 1920. (From the Company.)

Permutit Company, General Bulletin No. 101. New York City, 1920. (From the Company.)

Philippine Bureau of Civil Service, Twentieth Annual Report. Manila, 1920. (From the Company.)

Philippine Director of Education, Twentieth Annual Report. Manila, 1920. (From the Director.)

Portland Cement Association, Concrete Work in Cold Weather. Chicago, Ill., 1920. (From the Association.)

Pratt and Whitney Company, Precision Machine Tools. Hartford, Conn., 1920. (From the Company.)

Reed Prentice Company, Lathes and Radial Drilling Machines. Worcester, Mass., 1920. (From the Company.)

Reliance Spring and Manufacturing Company, Booklet for Every Purpose. Brooklyn, New York, 1920. (From the Company.)

Rhode Island Public Utilities Commission, Eighth Annual Report, 1919. Providence, R. I., 1920. (From the Commission.)

Smith, T. L., Company, Bulletin Nos. 405, 406, 409A, and 410. Milwaukee, Wisconsin, 1920. (From the Company.)

Smithsonian Institution, Annual Report for 1918. Washington, D. C., 1920. (From the Institution.)

South Bend Lathe Works, Text-Book No. 4. South Bend, Indiana, 1920. (From the Works.)

Southern Railway Company, Twenty-sixth Annual Report, Richmond, Va., 1919. (From the Company.)

Sprague Electric Works, Bulletin No. 44553.1. New York City, N. Y., 1920. (From the Works.)

Staffords National Service, Who's Who in Philadelphia in Wartime. Philadelphia, Pa., 1920. (From the Editor.)

Standard Scale and Supply Company, Catalog No. Y.190. Pittsburgh, Penna., 1920. (From the Company.)

Stewart, Frank H., Electric Company, Catalogue No. 22. Philadelphia, Penna., 1920. (From the Company.)

Strong, Carlisle and Hammond Company, Catalogue No. 22, Section C, and Catalogue No. 8-G. Cleveland, Ohio, 1920. (From the Company.)

Temple University, Annual Catalogue, 1920. Philadelphia, Pa. (From the University.)

Triumph Ice Machine Company, Bulletin 516, Cincinnati, Ohio, 1920. (From the Company.)

Truscon Steel Company, Buildings and Steel Windows. Youngstown, Ohio, 1920. (From the Company.)

Uehling Company, Bulletin 111, New York City, N. Y., 1920. (From the Company.)

United States Commissioner of Patents, Annual Report for 1919. Washington, D. C., 1920. (From the Commissioner.)

U. S. Department of Commerce, Commerce and Navigation of the United States for 1919. Washington, D. C. (From the Department.)

University of Nebraska, Fiftieth Annual General Catalogue. Lincoln, Nebraska, 1920. (From the University.)

Wahl, H. R., Company, Bulletin D-1. Chicago, Illinois, 1920. (From the Company.)

Waltham, Massachusetts, Second Inaugural Address of Hon. George R. Beal, Mayor, and Annual Reports for 1919. Waltham, Massachusetts, 1920. (From the City of Waltham.)

Whitcomb Blaisdell Machine Tool Company, Lathes, Planers. Worcester, Mass., 1920. (From the Company.)

Wisconsin Electric Company, Time and Labor Saving Devices. Racine, Wisconsin, 1920. (From the Company.)

## BOOK NOTICES.

CATALYSIS AND ITS INDUSTRIAL APPLICATIONS. By E. Jobling, A.R.S.C., B.Sc., A.I.C. Second edition, small 8vo, viii-134 pages, index and 12 illustrations. Philadelphia, P. Blakiston's Son & Co. $3.75, net.

Since 1836 when Berzelius coined the term catalysis for a series of mysterious chemical actions, the problems presented by these actions have been the subject of continual and extensive investigations, and a vast number of phenomena added to the few that were known to the Swedish chemist. As a matter of fact, a catalytic action, still practically unexplained, though very familiar, was announced in 1832, when Döbereiner published in the first number of the *Annalen*, his discovery of the peculiar action of manganese dioxide in determining the decomposition of potassium chlorate.

A large literature, most of it summarized in books, now exists concerning all types of these actions. The book in hand is a brief treatment of some of the more important industrial applications of the principals of catalytic action. Fifteen pages are devoted to a discussion of the general conditions favoring and interfering with these actions. The formal definition given is that of Ostwald: "A catalytic agent is that material which affects the velocity of a chemical reaction without appearing in the final products." Another definition, somewhat more pedantic perhaps, is: "A catalyst is a substance that increases the specific velocity of a reaction without stoichi-

ometric participation therein." It is pointed out in the book that a catalyst sometimes undergoes a physical change in the course of its action, the instance given being that of manganese dioxide which passes from a crystalline to an amorphous condition when used in the preparation of oxygen from potassium chlorate, but in this case the material is strongly heated, and this may be the cause as much as any specific action as a catalyst.

Many of the common industries are now dependent on catalytic methods for proper and economic operation, and the more important of these are described in concise and convenient form. The general style, printing and arrangement of the book are commendable.

HENRY LEFFMANN.

DICTIONARY OF EXPLOSIVES. By Arthur Marshall, A.C.G.I., F.I.C., F.C.S., Chemical Inspector, India Ordnance Department. 8vo. xiv–159 pages. Philadelphia, P. Blakiston's Son & Co. $3.75, net.

The extensive experience of the author of this book and his success in presenting the literature of it in highly satisfactory form, prevents any doubt as to the value of the present compilation. It presents brief statements of the composition of the several hundred explosives that are now known, enumerated alphabetically by their trade names and also classified according to their uses. The classes are: coalmine, blasting, high, miscellaneous and propelling. The resources of language have been strongly drawn on for the many names that have been coined to indicate the changes that have been rung on the comparatively few mixtures and compounds that are available for use as explosives. The reviewer has been a little puzzled by an allusion to an explosive termed "imperialite," the name being derived from that of the Marquis Imperiali, who used his private means in experimenting on explosives. He started a factory for the manufacture of some, and on the first day of its operation an explosion killed five workers. He rebuilt the factory and the day after it was re-started it blew up again, and the marquis was killed. The puzzling item in the account is that Marshall states that its history is that of the explosive that we meet "in the comic papers." There must be a joke here, but the reviewer is not "on to it."

The book contains a large amount of information, and will be of much use to chemists who cannot afford the more expensive works on the subject. The reviewer is moved to ask why there is no recent authoritative and comprehensive work on explosives emanating wholly from American sources, especially from the experts of the great du Pont works.

HENRY LEFFMANN.

NOTES ON CHEMICAL RESEARCH: AN ACCOUNT OF CERTAIN CONDITIONS WHICH APPLY TO ORIGINAL INVESTIGATION. By W. P. Dreaper, O.B.E., F.I.C. Second edition, small 8vo, xv–195 pages, no index. Philadelphia, P. Blakiston's Son & Co. $2.50, net.

As the author said in the preface to the first edition, modern science is based on the record of past investigation. He might have added that a feeling of gratitude should actuate all who practice a profession or a science which will induce each one to labor as far as in him lies to increase the stock of

knowledge. The bulk of the book is devoted to a free and extensive presentation of the methods of research and the apparatus to be used, taking the word in its widest sense to mean the literature and laboratory facilities. There is, as might be expected, a good deal of philosophic discussion which represents the author's views, entitled to serious consideration for he has given much thought to the subject, but concerning which there is room for difference of opinion.

The reviewer is most interested in the chapter on the course of training for the student, and hoped that some attention would be given to the earlier training, especially an expression of opinion as to the comparative value of the old university course in the so-called "humanities" and the modern so-called "practical" course. Dreaper, however, considers only that part of the students' training which concerns the later work, when specializing in the science. He refers to the "Heuristic" method of teaching without giving any details. The reviewer confesses ignorance of this method, but does not suppose that it consists in studying the properties of matter while bathing, as the word suggests. The author says that the period between the ages of 17 and 23 is a critical one. Unquestionably, but so is the period between 12 and 17. "Just as the twig is bent, the tree's inclined." Nothing is said about the training in language. The reviewer believes in the study of the classics in school life and the study of French and German in college life for all students intending scientific work, especially physics and chemistry.

With the great mass of Mr. Dreaper's statements, the reviewer has no quarrel.

HENRY LEFFMANN.

A TEXT-BOOK OF ORGANIC CHEMISTRY. By E. DeBarry Barnett, B.Sc. (Lond.), A.I.C. Lecturer in Organic Chemistry, Sir John Cass Technical Institute. Small 8vo, xii–369 pages, index and 15 illustrations. Philadelphia, P. Blakiston's Son & Co. $5.00, net.

A neatly printed book on good paper giving a survey of the field of organic chemistry in the form now usual in smaller manuals. It opens with the statement that "organic chemistry is best defined as the chemistry of the compounds of carbon" and proceeds to point out that the substances included in descriptive organic chemistry were called organic because they were regarded as formed only under the influence of "vital force," but that this idea was exploded when Wöhler, in 1828, obtained urea from inorganic sources. This is the usual phraseology. It is not quoted here to disapprove it, but to suggest that it would be an interesting matter to investigate the origin of the application of the term organic to a special field of chemistry. It is hardly true that the basis of the division of chemistry into its two familiar branches was "completely exploded" by Wöhler's discovery. Every chemist feels that some line of demarkation exists between the mineral kingdom and the living one, though it is true that in the highest levels of philosophy the unity of nature impresses itself upon us and we realize that "all are but parts of one stupendous whole." Indeed, the author says that the division is merely "a matter of convenience."

So far as the mass of text of the book is concerned, no unfavorable

criticism is to be recorded. A very large amount of information is presented in a convenient and vivid form, and the work will serve very well the purposes of the beginner in the study of its field.

One of the most interesting and striking features of the book is the advice as to the more elaborate literature to be consulted. About four pages are devoted to this phase, the greater portion of it to recommending German books, some in English translation but many in the original. It is doubtful if a book published in the United States would venture on this method. England has suffered vastly more than the United States from German methods of warfare, yet English scientists have never lost their heads and have always recognized the value of German work, especially in chemistry.

HENRY LEFFMANN.

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS.

Report No. 82, Airplane Stress Analysis. Preprint from Fifth Annual Report, 70 pages, illustrations, diagrams, quarto. Washington, Government Printing Office, 1920. This is a handbook of airplane stress analysis and was prepared in 1917, for the Chief Constructor of the Navy, Admiral D. W. Taylor, U. S. N. It was primarily intended to aid those who had to make for the U. S. Government estimates of airplane stress and factors of safety. The text, written by Dr. A. F. Zahm, comprises three parts: Part I, being introductory, presents briefly the mechanics of materials and of elementary structural members; Part II treats of airplane wing stresses; Part III treats of body stresses. Part IV, written by Mr. L. H. Crook, gives in detail the numerical solutions of the illustrative practical problems presented in the text, and by use of the methods and equations therein formulated. Finally, in Part V, prepared by the authors jointly, are assembled the tables, formulas and descriptive diagrams illustrating the text; also the graphical solutions of many of the standard stress problems given in the text.

Report No. 89, Comparison of Alcogas Aviation Fuel with Export Aviation Gasoline, 14 pages, illustrations, quarto. Washington, Government Printing Office, 1920. Aviation alcogas, prepared by the Industrial Alcohol Co., of Baltimore, Md., for trial by the Navy Department and by the latter submitted to the Bureau of Standards for test, was a mixture apparently of about 40 per cent. alcohol, 35 per cent. gasoline, 17 per cent. benzol, and 8 per cent. other ingredients. This is not the alcogas prepared for commercial or passenger car use. The exact composition and methods of manufacture are a trade secret. The tests made for the Navy Department consisted in a direct comparison, in a 12-cylinder Liberty engine, between alcogas and standard "X" (export grade) aviation gasoline with respect to maximum power attainable, and fuel consumption with the leanest mixture giving maximum power. The tests were made in the altitude laboratory at the Bureau of Standards, where controlled conditions simulate those of any altitude up to 30,000 feet. The speed range covered was from 1400 to 1800 revolutions per minute and the altitude range from ground level to 25,000 feet. Two series of comparisons were made, one with 5.6 compression ratio pistons and one with 7.2 compression ratio pistons.

## PUBLICATIONS RECEIVED.

*U. S. War Department:* Annual Reports, 1919, 6 volumes, illustrations, plates, maps, 8vo. Washington, Government Printing Office, 1919–1920. Contents: Vol. I, Part I.—Reports of the Secretary of War; Chief of Staff; Adjutant General; Commander-in-Chief, American Expeditionary Forces; Director General; Judge Advocate General, Quartermaster General, Chief Signal Officer. Vol. I, Parts II and III.—Report of the Surgeon General. Vol. I, Part IV.—Reports of the Chief of Ordnance; Chief of Militia Bureau; Chief of Coast Artillery; Chief of Construction Division; Director of Tank Corps; Director of Chemical War Service; Director of Real Estate Service, Board of Ordnance and Fortification; Director of National Defense; Chief of Motor Transport Corps; Chief of Transportation Service; Chief of Field Artillery; Military Parks. Vol. II.—Reports of the Chief of Engineers. Vol. III.—Reports of the Chief, Bureau of Insular Affairs, 1919; Governor of Porto Rica 1919; Philippine Commission 1918.

*U. S. Department of Mines:* Bulletin No. 184, The Manufacture of Sulphuric Acid in the United States, by A. E. Wells and D. E. Fogg, 216 pages, illustrations, plates, 8vo. Washington, Government Printing Office, 1920. Monthly Statement of Coal-Mine Fatalities in the United States, June and July, 1920, by W. W. Adams, 2 pamphlets, 8vo. Technical Paper 259, Production of Explosives in the United States during the Calendar Year, 1919, with notes on coal-mine accidents due to explosives and list of permissible explosives tested to May 31, 1920, by William W. Adams, 31 pages, illustrations, 8vo. Washington, Government Printing Office, 1920. Technical Paper 266, Coke-oven Accidents in the United States during the Calendar Year, 1919, by William W. Adams, 25 pages, 8vo. Washington, Government Printing Office, 1920.

*Ontario Department of Mines:* Bulletin No. 882, 29th Annual Report, being Vol. XXIX, Part III, 1920, 95 pages, illustrations, plates, maps, 8vo. Toronto, King's Printer, 1920.

*U. S. Department of Agriculture:* Bulletin No. 882, Manufacturing and Laboratory Tests to Produce an Improved Cotton Airplane Fabric, by Fred Taylor and D. E. Earle, 48 pages, illustrations, 8vo. Washington, Government Printing Office 1920.

*Pratt Institute Free Library:* Technical Books of 1919. A selection, 28 pages, 16mo. Brooklyn, N. Y., Pratt Institute Free Library, 1920.

*Ajax Metal Company:* Ajax Metal Products: White Metal Alloys, Bronze and Brass Ingots, Bearings and Castings. Philadelphia, Ajax Metal Company, 1920. 38 pages, illustrations, quarto.

---

**Atomic Weight of Thulium.**—C. JAMES and O. J. STEWART of New Hampshire College have redetermined the atomic weight of thulium, and obtained a value of 169.44 (average of five determinations). The ratio determined was that of thulium chloride $TmCl_3$ to silver (*Jour. Am. Chem. Soc.*, 1920, xlii, 2022–2023).

J. S. H.

# CURRENT TOPICS.

**The National Advisory Committee for Aeronautics** has just issued a bulletin received from its Paris office, in which the conditions and possibilities of commercial air service are extensively discussed. The air offers some great advantages as a commercial route. No survey nor construction work is needed, nor are land damages or rights of way to be considered, unless, indeed, legislation should take a hand in the matter. Grade-crossing accidents, head-on and tail-end collisions are not impossible, but reasonable care will avoid such, for air lanes may be established similar to sea lanes that have been laid down for transatlantic travel.

According to the aeronautical correspondent of the *London Times*, a question asked in the House of Commons brought out a statement that within twelve weeks over 2000 passengers have been carried by airplane in England, and that the service between London and the Continent is now being operated with the regularity of an express train, and has realized lately a small profit.

Mr. Frank Searle, General Manager of the Airco Company, presents in the bulletin under consideration a review of the problems of regular air transport. The Airco Company can be considered a real commercial aerial transportation company, as it carries about 1000 passengers per month between London and Paris. There are many problems to be solved before commercial airplane service can be placed on a basis as safe and satisfactory as existing methods of transportation.

Among the serious difficulties with the early airplanes was the need of constant attention and repair. Looked at from a commercial point of view, these older airplanes were capable of earning money only about an hour and a half during the twenty-four, the rest of the time the machine was in its shed, and insurance and overhead charges were running on without break. To-day the machine is called on to do five times more work than formerly. No data are at hand as yet by which to determine exactly the average life of the commercial aero-engine, but there are motor omnibuses now running satisfactorily in London, the engines of which are ten years old. If these can stand the strains of travel, it seems reasonable to suppose that engines can be constructed for airplanes which will have at least as long a life. The absence of grinding wear and tear, so serious and inevitable a condition of earth transport, stands out conspicuously as a factor in the London-Paris air service. It could not be expected that the comparatively frail construction of these planes could stand for a day's rough and tumble work, but they rush to and fro in the air

without any appreciable damage for a limited time. Small renewals and repairs are constantly required, but these are not of serious moment in connection with the problem. Repairing by night seems to offer especial advantages. If a commercial machine is scheduled to do five hours work per day, there will be nineteen hours for keeping it in trim. A night shift of mechanics now works regularly at Croydon.

The speed of flying machines is an important factor in determining their practical value. A transporting apparatus which can easily make 100 miles per hour, taking only five hours to make a round trip which with boat or train would require twenty hours, is in a field by itself.

Designs and construction of commercial machines will have to be made matters of specific engineering investigation. The problem is different from that presented in war or sport. The question of overhaul and repair must be kept always in view. Take the question of motor, for example. If the motor of a given machine needs repair, it will be a great waste if the whole machine must go into the shop for some days. A reserve of engines should be provided, and the planes so constructed that with but little delay the substitution can be made. A commercial plane must therefore be designed which will permit of the whole motor being taken out without difficulty.

Mr. Searle thinks that the fast passenger traffic will be soon represented by two types of craft. One will be a simplified plane, somewhat of the type in use on the London-Paris service known as "Airco 16." The other should be larger, with capacity, say, for eight persons. Both, he thinks, should be single engine machines.

The latter view is a matter of controversy, but it seems certain that, from the point of view of handling and maintenance, the commercial one-motor is preferable. One point, however, is that of safety, for a two-motor machine may avoid a forced landing. Mr. Searle's answer to this is that the single engine should be constructed like the "wonderful one-hoss shay," it may wear out but can never break down. In exemplification of his views, Mr. Searle refers to the experiences with engines now used on the London-Paris service. Much of the trouble with them is due to the gears between the engine and propeller, which, for good aerodynamic reasons, it is true, cause the propellers to revolve at a lower speed than the engine. Such gears, Mr. Searle advises, should be eliminated, and a direct drive installed. There are, indeed, all sorts of minor troubles connected with the existing installation of motors in airplanes, but the plane taken as a whole, as a machine developing a certain amount of power, very rarely gives trouble when it is in proper trim. When the detailed improvements indicated by modern experience have been made, the chance of a breakdown should be remote, and the single engine machine should, for some time at least, run for short stages with

a risk of failure so low as to be practically negligible. It is unwise to make great sacrifices to lightness. The fundamental point is that the motor has to keep running to earn the money required. Therefore, even if it is a little on the heavy side, the manager will be content if it will do its work with the least trouble and overhauling. The Continental air-express service has a staunchly built six-cylinder motor, which stands up well to its work; in the words of one pilot, "it practically never lets one down." Big aero-motors are now wanted, developing a lot of power and doing it as a routine matter. Above all, they must be simple, readily accessible in all their working parts and easy to install and remove.

Much discussion has been held lately as to the substitution of metal for wood in airplane wings. This substitution should be made slowly, but it seems likely that an increasing use of metal will be noted as time goes on.

An international system by which the methods of packing and the types of machines are uniform will be necessary before extensive substitution of airplane transport of freight can compete with the methods now in use.

The question of transport over boundaries will be one of great importance and difficulty. All surface systems are easily controlled by the countries through which they pass, but air travel is far less easily supervised. Low flying machines are generally amenable to control, but in fogs air transport may be carried on without any suspicions by those on the land, and it will not be long before very high flights are possible, and these can be made at night at high speed, so as to defy any present methods of supervision. Night-flying service is, indeed, being seriously discussed, but not a secret one. It is proposed to have the lines of travel indicated by special lights, so that a pilot, flying low, can pick his way and be helped in adhering to the compass course. There are now methods known for the construction of lights that will act automatically for long periods, lighting themselves at night and extinguishing themselves in the morning.

All machines should be fitted with wireless telephone service, which will be of great use in passing through fog. Aviators frequently soar above the fog and steer by means of the compass alone.

There is little doubt that before long the distances between important points will be reduced by fast air-service to very small amounts expressed in hours. H. L.

**The Valency of Photo-electrons and the Photo-electric Properties of Some Insulators.** M. J. KELLY. (*Phys. Rev.*, October, 1920.)—The method employed as long ago as 1911 by Colonel Millikan for studying the charges upon droplets was again used. The two horizontal plates of a condenser are about 1.5 cm. apart. The air between them is enclosed and is maintained at a constant temperature in order to avoid convection currents. Sulphur was

heated to 150° C. in a special atomizer. At that temperature it is a pale yellow liquid and can be easily atomized. By the instrument a group of droplets is produced. These fall owing to their weight, and some pass through holes in the upper condenser plate, thus getting into the air space between the two plates of the condenser. Here they are illuminated and are observed through a telescope with high magnification. The observer selects a suitable drop and follows its motion. It falls with very small speed. If, however, it has an electric charge and if in addition the two condenser plates are oppositely charged, then, besides its weight, another force acts on it, *viz.*, the force resulting from the interaction of its charge with the field of force between the plates. It may go up instead of falling as before and in general its speed will be different. From a knowledge of the constants of the drop, of its speed, and of the difference of potential between the plates it is possible to calculate the electrical charge on the drop. The author used a P.D. of about 7000 volts.

In the case of sulphur the drops were always found to have a static charge. They sublimed rapidly, so rapidly that a drop after 30 minutes required 28 seconds to fall just as far as it fell in 8.2 seconds at the beginning of the series of observations. The lengthening of the time was due to the dwindling size of the sphere. To avoid this error the walls of the enclosed chamber were encrusted with flowers of sulphur. After several days the air attained a relative humidity of 100 per cent. in terms of sulphur vapor and the drops could no longer evaporate.

The motion of a droplet was studied, then the observer opened a shutter and let a beam of ultra-violet light fall on it until a change of speed showed that one electron or more had been emitted. The shutter was then closed and the speed of the drop again determined. The shutter was reopened, another change of speed obtained, and so on. "Some drops were kept under observation for two hours; .57 separate emissions were observed on one drop and over 500 emissions were observed under good experimental conditions and calculated." In all cases except two the emission was found to consist of a single electron. In the two unusual cases it consisted of two electrons. With shellac every one of the 800 emissions observed consisted of a single electron. "This is rather interesting, especially in the case of shellac in which there are complex molecules made up of a large number of atoms."

Previous investigations have shown that ionization by X-rays, by gamma rays, by beta and alpha particles, as well as in the photo-electric effect in mercury when acted on by ultra-violet light consists in the emission of a single elementary charge. The significance of Mr. Kelly's work lies in showing that in insulators where there is a dearth of free electrons (evidenced by the small electrical conductivity) the same rule holds that in ionization one electron, and one only, is set free. G. F. S.

**Electrification by Impact.** HAROLD F. RICHARDS. (*Phys. Rev.*, October, 1920.)—In the production of electricity by the rubbing of one substance by another not only is there intimate contact produced, but friction also comes into play. Owing, perhaps, to the superposition of these two actions, the data of frictional electricity are somewhat erratic and a consistent theory is still to be framed. The purpose of this investigation is to study the effect of intimate contact of a metal with an insulator with no complication from friction.

An ebonite disk with its surface horizontal was let fall upon a horizontal brass disk. The charge developed upon the latter by the impact was measured by a tilted Wilson electroscope. The brass and the ebonite disks were given surfaces as flat and as smooth as possible. Between impacts radium was used to discharge the ebonite disk. The brass always acquired a positive charge and these charges were of the same order of magnitude as those produced by friction. The ebonite disk fell from different heights and thus struck the brass cylinder while moving with different speeds. In general the higher this speed the greater the charge developed. It was found that after repeated impacts the charge at a certain velocity was greater than at the beginning of the series of experiments. This may well have been due to the flattening out of the ebonite surface by which more intimate contact was produced. A variable condenser was joined to the brass anvil and determinations of the charges developed were made when the capacity varied. The charge was found to depend in no way on the capacity of the brass disk. Furthermore, the greater the weight on the falling disk, the greater the charge produced. When the upper disk was lowered gently upon the lower one a slight charge of .02 E.S.U. was caused. The addition of as much as 25 pounds to the ebonite disk while it was resting on the brass caused no increase in this small charge.

A second set of experiments was carried out in which an ebonite sphere was hurled vertically upward and struck the lower surface of a brass disk. The connection between speed of impact and charge developed is similar to that holding in the case of the impact of two disks. A glass ball and a zinc plate were also used in all combinations. There seems to be no relation between the electrical energy developed and the mechanical energy lost during impact.

A thorough revision of the experimental data of frictional electricity is needed in order to disentangle the discordant and contradictory congeries of observations made during the last two centuries. Mr. Richards has made a very promising beginning in this work. G. F. S.

PRESS OF
J. B. LIPPINCOTT COMPANY
PHILADELPHIA

# INDEX TO VOL. 190.


ADAMS, L. H., and E. D. WILLIAMSON: The annealing of glass, 597, 835

Aerodynamic balance (Zahm), 685

Air pump, Priestley (Sellers), 877

Ajax-Wyatt furnace, Motor effect in the (Northrup), 817

Alcohol, Industrial (Tunison), 373

Alternating currents for submarine cable transmission (Pernot), 323

Annealing of glass (Adams and Williamson), 597, 835

ARRHENIUS, SVANTE AUGUST: The problem of the world's supply of energy, 123

Artillery equipment used by the American Expeditionary Forces in France (Williams), 301

Audion, The (De Forest), 1

BLATTERMAN, A. S.: Theory and practical attainments in the design and use of radio-direction finding apparatus using closed coil antennas, 421

BOOK NOTICES:

Andros, S. O.: Fuel oil in industry, 587

Barnett, E. deB.: A text-book of organic chemistry, 911

Brown, J. C.: A history of chemistry from the earliest times. Second edition, 448

Dolt, M. L.: Chemical French, 588

Dreaper, W. P.: Notes on chemical research, 910

Friend, J. N.: A text-book of inorganic chemistry, vol. ix, part 1, 144

Huyghens, C.: Traité de la lumiére, 280

Jobling, E.: Catalysis, second edition, 909

Lavoisier, A. L.: Mémoires sur la respiration et la transpiration des animaux, 447

Leeds and Northrup Company, Electrometric methods and apparatus, for determining hydrogen-ion concentrations, 589

Lempfert, R. G. K.: Meteorology, 751

Marshall, A.: Dictionary of explosives, 910

Molinari, E.: Treatise on general and industrial inorganic chemistry, second edition, 145

National Advisory Committee for Aeronautics, Reports, 146, 280, 449, 589, 752, 912

Spallanzani, L.: Observations et expériences faites sur les animalcules des infusions, 447

BULLOCK, E. R., and J. G. CAPSTAFF: A production of panchromatic sensitiveness without dyes, 871

Cable transmission, Submarine, The use of alternating currents for (Pernot), 323

CAPSTAFF, J. G., and E. R. BULLOCK: A production of panchromatic sensitiveness without dyes, 871

Chemistry of the earth's crust (Washington), 757

CLAMER, G. H.: Induction electrical furnace, 473

Colorimeter Schreiner, Simple changes which effect a marked improvement in the (Davis), 243

CURRENT TOPICS: 121, 122, 126, 132, 136, 149, 150, 185, 209, 258, 262, 266, 269, 270, 283, 284, 322, 371, 372, 422, 430, 434, 438, 451, 508, 565, 566, 569, 570, 572, 576, 591, 592, 632, 656, 684, 725, 732, 735, 739, 752, 753, 816, 870, 874, 875, 880, 890, 893, 894, 896, 900, 913, 914

DAVIS, C. W.: Simple changes which effect a marked improvement in the Schreiner colorimeter, 243
DAY, ARTHUR L.: Optical glass and its future as an American industry, 453
DE FOREST, LEE: The audion—its action and some recent applications, 1
DEISCH, NOEL, and LOYD A. JONES: The measurement of graininess in photographic deposits, 657
Design of war vessels as affected by the world war (Taylor), 157

Earth's crust, The chemistry of the (Washington), 757
Eastman Kodak Company: Research Laboratory, Notes, 127, 259, 891
Electrical furnace, induction (Clamer), 473
Electrical furnace, Nature and explanation of the "motor effect" in the Ajax and Wyatt furnace (Northrup), 817
Energy, World's supply of (Arrhenius), 114

FIELDNER, A. C., S. H. KATZ, S. P. KINNEY, and E. S. LONGFELLOW: Poisonous gases from carbon tetrachloride fire extinguishers, 543
Fire extinguishers, Poisonous gases from carbon tetrachloride (Fieldner, Katz, Kinney and Longfellow), 543
FISCHER, HERMAN EDWARD: The nitrogen industry, 187
FRANKLIN INSTITUTE:
Franklin Medal presentation, 91
Library Notes, 139, 271, 440, 581, 745, 905
Membership Notes, 137, 271, 439, 577, 744, 904
Science and Arts Committee, Abstracts of proceedings of meetings: June 2, 1920, 137; September 1, 1920, 577; October 6, 1920, 743; November 3, 1920, 903
Sections, Proceedings of meetings, 743, 903
Stated Meetings, Proceedings: October 20, 1920, 741; November 17, 1920, 901

Gamma rays, Scattering of (Gray), 633
Gases, Poisonous, from carbon tetrachloride fire extinguishers (Fieldner, Katz, Kinney and Longfellow), 543
Glass, Annealing of (Adams and Williamson), 597, 835
Glass, Optical (Day), 453
Gravitation and relativity (Kanolt), 211
GRAY, J. A.: Scattering of X- and γ-rays, 633

Induction electrical furnace (Clamer), 473
Industrial alcohol (Tunison), 373

JONES, L. A.: On the theory of tone reproduction with a graphic method for the solution of problems, 39
JONES, LOYD A., and NOEL DEISCH: The measurement of graininess in photographic deposits, 657

KANOLT, C. W.: Gravitation and relativity, 211
KATZ, S. H., A. C. FIELDNER, S. P. KINNEY and E. S. LONGFELLOW: Poisonous gases from carbon tetrachloride fire extinguishers, 543
KINNEY, S. P., A. C. FIELDNER, S. H. KATZ and E. S. LONGFELLOW: Poisonous gases from carbon tetrachloride fire extinguishers, 453

LICHTENBERG, CHESTER: U. S. Army searchlights, 509, 691
LONGFELLOW, E. S., A. C. FIELDNER, S. H. KATZ and S. P. KINNEY: Poisonous gases from carbon tetrachloride fire extinguishers, 543

Nela Research Laboratory, Notes, 257, 429
Nitrogen industry (Fischer), 187
NORTHRUP, EDWIN F.: Nature and explanation of the "motor effect" in the Ajax-Wyatt furnace, 817

Optical glass and its future as an American industry (Day), 453

Panchromatic sensitiveness, A production of, without dyes (Capstaff and Bullock), 871
PARSONS, CHARLES A.: Some reminiscences of early days of turbine development, 111
PERNOT, FREDERICK E.: The use of alternating currents for submarine cable transmission, 323
Photographic deposits, The measurement of graininess in (Jones and Deisch), 657
Photography, On the theory of tone reproduction (Jones), 39
Poisonous gases from carbon tetrachloride fire extinguishers (Fieldner, Katz, Kinney and Longfellow), 543
Priestley air pump (Sellers), 877
Publications received: 148, 282, 450, 591, 913

Radio-direction finding apparatus, Theory and practical attainments in the design and use of (Blatterman), 421
Relativity, Gravitation and (Kanolt), 211

Scattering of X- and γ-rays (Gray), 633
Schreiner colorimeter, Simple changes which effect a marked improvement in the (Davis), 243
Searchlights, U. S. Army (Lichtenberg), 509, 691
SELLERS, COLEMAN, JR.: The Priestley air pump, 877
Sensitiveness, panchromatic, A production of, without dyes (Capstaff and Bullock), 871

TAYLOR, DAVID WATSON: The design of war vessels as affected by the world war, 157
Tone reproduction, On the theory of (Jones), 39
TUNISON, BURNELL R.: Industrial alcohol, 373
Turbine development, Some reminiscences of early days of (Parsons), 111

U. S. Army searchlights (Lichtenberg), 509, 691
U. S. Bureau of Chemistry, Notes: 129, 263, 431, 571, 733, 895
U. S. Bureau of Mines, Notes: 133, 267, 435, 573, 737, 897
U. S. Bureau of Standards, Notes: 123, 245, 423, 567, 727, 881

War vessels, Design of (Taylor), 157
WASHINGTON, HENRY S.: The chemistry of the earth's crust, 757
WILLIAMS, C. C.: Types of artillery equipment used by the American Expeditionary Forces in France, with a brief discussion of their uses and the reasons for their adoption, 301
WILLIAMSON, E. D., and L. H. ADAMS: The annealing of glass, 597, 835
Wind-balance, The cross-arm (Zahm), 685
World's supply of energy, The problem of the (Arrhenius), 114
Wyatt furnace, Motor effect in the (Northrup), 817

X-rays, Scattering of (Gray), 633

ZAHM, A. F.: The cross-arm wind-balance, 685